Random Walks and Resistance

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Based on a handout by Aaron Anderson

Instructor's Handout

This file contains solutions and notes.
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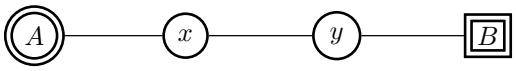
Part 1: Random Walks

Consider the graph below. A particle sits on some node n. Every second, this particle moves left or right with equal probability. Once it reaches node A or B, it stops.
We would like to compute the probability of our particle stopping at node A.

In other words, we want a function $P : \text{Nodes} \to [0, 1]$ that maps each node of the graph to the probability that our particle stops at A.



Problem 1:
What are $P(A)$ and $P(B)$ in the graph above?
Note that these values hold for all graphs.

Solution

$P(A) = 1$ and $P(B) = 0$

Problem 2:
Find an expression for $P(x)$ in terms of $P(y)$ and $P(A)$.
Find an expression for $P(y)$ in terms of $P(x)$ and $P(B)$.

Solution

$P(x) = \frac{P(A)+P(y)}{2}$

$P(y) = \frac{P(B)+P(x)}{2}$

Problem 3:
Use the previous problems to find $P(x)$ and $P(y)$.

Solution

$P(x) = 2/3$

$P(y) = 1/3$