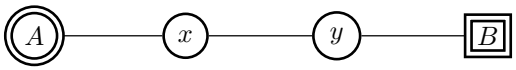
Random Walks and Resistance

Prepared by Mark on May 24, 2024
Based on a handout by Aaron Anderson

Part 1: Random Walks

Consider the graph below. A particle sits on some node n. Every second, this particle moves left or right with equal probability. Once it reaches node A or B, it stops.
We would like to compute the probability of our particle stopping at node A.

In other words, we want a function $P : \text{Nodes} \to [0, 1]$ that maps each node of the graph to the probability that our particle stops at A.



Problem 1:
What are $P(A)$ and $P(B)$ in the graph above?
Note that these values hold for all graphs.

Problem 2:
Find an expression for $P(x)$ in terms of $P(y)$ and $P(A)$.
Find an expression for $P(y)$ in terms of $P(x)$ and $P(B)$.

Problem 3:
Use the previous problems to find $P(x)$ and $P(y)$.