Definition 1.23 (Ordered pair). $\langle a, b \rangle = \{\{a\}, \{a, b\}\}$.

Having fixed a definition of an ordered pair, we can use it to define further sets. For example, sometimes we also want ordered sequences of more than two objects, e.g., *triples* $\langle x, y, z \rangle$, *quadruples* $\langle x, y, z, u \rangle$, and so on. We can think of triples as special ordered pairs, where the first element is itself an ordered pair: $\langle x, y, z \rangle$ is $\langle \langle x, y \rangle, z \rangle$. The same is true for quadruples: $\langle x, y, z, u \rangle$ is $\langle \langle \langle x, y \rangle, z \rangle, u \rangle$, and so on. In general, we talk of *ordered n-tuples* $\langle x_1, \dots, x_n \rangle$.

Certain sets of ordered pairs, or other ordered n-tuples, will be useful.

Definition 1.24 (Cartesian product). Given sets A and B, their *Cartesian product* $A \times B$ is defined by

$$A \times B = \{\langle x, y \rangle : x \in A \text{ and } y \in B\}.$$

Example 1.25. If $A = \{0, 1\}$, and $B = \{1, a, b\}$, then their product is

$$A \times B = \{\langle 0, 1 \rangle, \langle 0, a \rangle, \langle 0, b \rangle, \langle 1, 1 \rangle, \langle 1, a \rangle, \langle 1, b \rangle\}.$$

Example 1.26. If A is a set, the product of A with itself, $A \times A$, is also written A^2. It is the set of *all* pairs $\langle x, y \rangle$ with $x, y \in A$. The set of all triples $\langle x, y, z \rangle$ is A^3, and so on. We can give a recursive definition:

$$\begin{aligned} A^1 &= A \\ A^{k+1} &= A^k \times A \end{aligned}$$

Proposition 1.27. *If A has n elements and B has m elements, then $A \times B$ has $n \cdot m$ elements.*