

| EXACT VALUES FOR THE SIN OF ALL INTEGERS...BY JAMES T. PARENT | |
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| 0 | 0 |
| 1 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 2 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 3 | $\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}$ |
| 4 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 5 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1-\frac{1}{3}\sqrt{3}\right)+\frac{1}{32}I\sqrt{-6\left(1-\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1-\frac{1}{3}\sqrt{3}\right)-\frac{1}{32}I\sqrt{-6\left(1-\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)$ |
| 6 | $\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}$ |
| 7 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 8 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 9 | $\frac{1}{8}\sqrt{2}(1+\sqrt{5})-\frac{1}{8}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}$ |
| 10 | $-\frac{1}{8}(-4+4I\sqrt{3})\left(\frac{1}{3}\right)(1+I\sqrt{3})-\frac{1}{8}(-4-4I\sqrt{3})\left(\frac{1}{3}\right)(1-I\sqrt{3})$ |
| 11 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})-\frac{1}{64}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})+\frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})-\frac{1}{64}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})+\frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 12 | $\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)$ |
| 13 | $\frac{1}{2}\text{sqrt}\left(2+2\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)\right)$ $+2\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 14 | $\frac{1}{2}\text{sqrt}\left(2+2\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)\right)$ $+2\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |

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| 15 | $\frac{1}{4}\sqrt{2}(\sqrt{3}-1)$ or $-\frac{1}{2}\left(\left(-\frac{1}{16}+\frac{1}{16}I\right)\sqrt{2}\right)^{\left(\frac{1}{3}\right)}(1+I\sqrt{3})-\frac{1}{2}\left(\left(-\frac{1}{16}-\frac{1}{16}I\right)\sqrt{2}\right)^{\left(\frac{1}{3}\right)}(1-I\sqrt{3})$ |
| 16 | $\frac{1}{2}\sqrt[3]{2-2\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ $-2\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ |
| 17 | $\frac{1}{2}\sqrt[3]{2-2\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ $-2\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ |
| 18 | $\frac{1}{4}\sqrt{5}-\frac{1}{4}$ |
| 19 | $\frac{1}{2}\sqrt[3]{2-2\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ $-2\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ |
| 20 | $-\frac{1}{8}(-4\sqrt{3}+4I)^{\left(\frac{1}{3}\right)}(1+I\sqrt{3})-\frac{1}{8}(-4\sqrt{3}-4I)^{\left(\frac{1}{3}\right)}(1-I\sqrt{3})$ |
| 21 | $\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})$ |
| 22 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}-\frac{1}{64}\sqrt{5}+\frac{1}{128}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}-\frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right.$ $\left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}+\frac{1}{8}\sqrt{5}-\frac{1}{16}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}+\frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\right.$ $\left.-\frac{1}{64}\sqrt{5}+\frac{1}{128}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}-\frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right.$ $\left.-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}+\frac{1}{8}\sqrt{5}-\frac{1}{16}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}+\frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right)^2+1}\right)^{\left(\frac{1}{3}\right)}$ |
| 23 | $\frac{1}{2}\sqrt[3]{2-2\left(-\frac{1}{16}\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)\sqrt{9-\sqrt{5}+\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}}\right.$ $\left.+\frac{1}{8}\sqrt{\frac{1}{4}\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2(9-\sqrt{5}+\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3})-1}\right)^{\left(\frac{1}{3}\right)}$ $-2\left(-\frac{1}{16}\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)\sqrt{9-\sqrt{5}+\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}}\right.$ $\left.-\frac{1}{8}\sqrt{\frac{1}{4}\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2(9-\sqrt{5}+\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3})-1}\right)^{\left(\frac{1}{3}\right)}$ |
| 24 | $\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}$ |
| 25 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1+\frac{1}{3}\sqrt{3}\right)+\frac{1}{32}I\sqrt{-6\left(1+\frac{1}{3}\sqrt{3}\right)^2+16}\right)^{\left(\frac{1}{3}\right)}$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1+\frac{1}{3}\sqrt{3}\right)-\frac{1}{32}I\sqrt{-6\left(1+\frac{1}{3}\sqrt{3}\right)^2+16}\right)^{\left(\frac{1}{3}\right)}$ |

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| 26 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(\frac{1}{64}-\frac{1}{64}\sqrt{5}-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}+\frac{1}{8}I\sqrt{-\left(-\frac{1}{8}+\frac{1}{8}\sqrt{5}+\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(\frac{1}{64}-\frac{1}{64}\sqrt{5}-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}-\frac{1}{8}I\sqrt{-\left(-\frac{1}{8}+\frac{1}{8}\sqrt{5}+\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 27 | $\frac{1}{4}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{2}(\sqrt{5}-1)$ |
| 28 | $\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right.$ $\left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)+\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})\right.$ $\left.-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 29 | $\frac{1}{2}\sqrt{2-2\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)}\left(\frac{1}{3}\right)$ $-2\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 30 | $\frac{1}{2}$ |
| 31 | $\frac{1}{2}\sqrt{2-2\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)}$ $\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)-$ $2\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)$ $\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 32 | $\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})+\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 33 | $\frac{1}{16}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})+\frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}$ |
| 34 | $\left(\frac{1}{64}-\frac{1}{64}\sqrt{5}-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}+\frac{1}{8}I\sqrt{-\left(-\frac{1}{8}+\frac{1}{8}\sqrt{5}+\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $+\left(\frac{1}{64}-\frac{1}{64}\sqrt{5}-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}-\frac{1}{8}I\sqrt{-\left(-\frac{1}{8}+\frac{1}{8}\sqrt{5}+\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 35 | $\left(-\frac{1}{32}\sqrt{6}\left(1+\frac{1}{3}\sqrt{3}\right)+\frac{1}{32}I\sqrt{-6\left(1+\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)+\left(-\frac{1}{32}\sqrt{6}\left(1+\frac{1}{3}\sqrt{3}\right)-\frac{1}{32}I\sqrt{-6\left(1+\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)$ |
| 36 | $\frac{1}{8}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}$ |

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| <p>37</p> | $\frac{1}{8}(-4\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} - 4\sqrt{2}\sqrt{5} + 4\sqrt{5}\sqrt{5+\sqrt{5}} - 4\sqrt{2} - 4\sqrt{5+\sqrt{5}} - 4\sqrt{2}\sqrt{5}\sqrt{3} - 4\sqrt{2}\sqrt{3} + 4\sqrt{5+\sqrt{5}}\sqrt{3}$ $+ 16I\sqrt{\sqrt{3}-\sqrt{5}\sqrt{3}-\sqrt{2}\sqrt{5+\sqrt{5}}+8}) \left(\frac{1}{3}\right) + \frac{1}{8}(-4\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} - 4\sqrt{2}\sqrt{5} + 4\sqrt{5}\sqrt{5+\sqrt{5}} - 4\sqrt{2} - 4\sqrt{5+\sqrt{5}} - 4\sqrt{2}\sqrt{5}\sqrt{3} - 4\sqrt{2}\sqrt{3}$ $+ 4\sqrt{5+\sqrt{5}}\sqrt{3} - 16I\sqrt{\sqrt{3}-\sqrt{5}\sqrt{3}-\sqrt{2}\sqrt{5+\sqrt{5}}+8}) \left(\frac{1}{3}\right)$ |
| <p>38</p> | $\left(-\frac{1}{64} - \frac{1}{64}\sqrt{5} + \frac{1}{128}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3} - \frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right.$ $+ \frac{1}{8}I\sqrt{-\left(\frac{1}{8} + \frac{1}{8}\sqrt{5} - \frac{1}{16}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3} + \frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right)^2 + 1} \left(\frac{1}{3}\right) + \left(-\frac{1}{64} - \frac{1}{64}\sqrt{5} + \frac{1}{128}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3}\right.$ $\left. - \frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} - \frac{1}{8}I\sqrt{-\left(\frac{1}{8} + \frac{1}{8}\sqrt{5} - \frac{1}{16}\sqrt{2}\sqrt{5+\sqrt{5}}\sqrt{3} + \frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right)^2 + 1} \left(\frac{1}{3}\right)\right)$ |
| <p>39</p> | $\frac{1}{32}\sqrt{2}(2\sqrt{3}(1+\sqrt{5}) + \sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}} - \sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}} + 2 + 2\sqrt{5})$ |
| <p>40</p> | $\frac{1}{4}\left(-\frac{1}{8}(-4\sqrt{3}+4I) \left(\frac{1}{3}\right) (1+I\sqrt{3}) - \frac{1}{8}(-4\sqrt{3}-4I) \left(\frac{1}{3}\right) (1-I\sqrt{3})\right)^2 \left(\frac{2}{3}\right) \left(\left(\sqrt{2}(-\sqrt{3}-1+I\sqrt{3}-I) \left(\frac{1}{3}\right) + (-\sqrt{2}(\sqrt{3}+1+I\sqrt{3}-I) \left(\frac{1}{3}\right))\right.\right.$ $\left.\left. \left(\sqrt{2}(-\sqrt{3}+1+I\sqrt{3}+I) \left(\frac{1}{3}\right) + (-\sqrt{2}(\sqrt{3}-1+I\sqrt{3}+I) \left(\frac{1}{3}\right))\right)\right)$ |
| <p>41</p> | $\frac{1}{2}\text{sqrt}\left(2 - 2\left(-\frac{1}{2} - \frac{1}{2}I\sqrt{3}\right)\right)$ $\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5}) + \frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}} + \frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5}) - \frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2 + 1} \left(\frac{1}{3}\right) - 2\right.$ $\left(-\frac{1}{2} + \frac{1}{2}I\sqrt{3}\right)$ $\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5}) + \frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}} - \frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5}) - \frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2 + 1} \left(\frac{1}{3}\right)\right)$ |
| <p>42</p> | $\frac{1}{32}((\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}} - \sqrt{2}(\sqrt{3}-1)(1+\sqrt{5}))\sqrt{\sqrt{3}\sqrt{5}-\sqrt{3}+\sqrt{2}\sqrt{5+\sqrt{5}}+8}$ |
| <p>43</p> | $\left(-\frac{1}{128}\sqrt{2}\sqrt{3} + \frac{1}{128}\sqrt{5+\sqrt{5}}\sqrt{3} - \frac{1}{128}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} - \frac{1}{128}\sqrt{5}\sqrt{5+\sqrt{5}} + \frac{1}{128}\sqrt{2}\sqrt{5} + \frac{1}{128}\sqrt{5+\sqrt{5}} - \frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3} + \frac{1}{128}\sqrt{2}\right.$ $+ \frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}\sqrt{3} - \frac{1}{16}\sqrt{5+\sqrt{5}}\sqrt{3} + \frac{1}{16}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} + \frac{1}{16}\sqrt{5}\sqrt{5+\sqrt{5}} - \frac{1}{16}\sqrt{2}\sqrt{5} - \frac{1}{16}\sqrt{5+\sqrt{5}} + \frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3} - \frac{1}{16}\sqrt{2}\right)^2 + 1} \left(\frac{1}{3}\right)$ $+ \left(-\frac{1}{128}\sqrt{2}\sqrt{3} + \frac{1}{128}\sqrt{5+\sqrt{5}}\sqrt{3} - \frac{1}{128}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} - \frac{1}{128}\sqrt{5}\sqrt{5+\sqrt{5}} + \frac{1}{128}\sqrt{2}\sqrt{5} + \frac{1}{128}\sqrt{5+\sqrt{5}} - \frac{1}{128}\sqrt{2}\sqrt{5}\sqrt{3} + \frac{1}{128}\sqrt{2}\right.$ $\left. - \frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}\sqrt{3} - \frac{1}{16}\sqrt{5+\sqrt{5}}\sqrt{3} + \frac{1}{16}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}} + \frac{1}{16}\sqrt{5}\sqrt{5+\sqrt{5}} - \frac{1}{16}\sqrt{2}\sqrt{5} - \frac{1}{16}\sqrt{5+\sqrt{5}} + \frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3} - \frac{1}{16}\sqrt{2}\right)^2 + 1} \left(\frac{1}{3}\right)\right)$ |

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| 44 | $\left(-\frac{1}{16} \left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right) \sqrt{9 - \sqrt{5} + \sqrt{2} \sqrt{5 + \sqrt{5}} \sqrt{3}} \right. \\ \left. + \frac{1}{16} I \sqrt{ -\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 (9 - \sqrt{5} + \sqrt{2} \sqrt{5 + \sqrt{5}} \sqrt{3}) + 4} \right) \left(\frac{1}{3} \right) + \left(-\frac{1}{16} \left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right) \sqrt{9 - \sqrt{5} + \sqrt{2} \sqrt{5 + \sqrt{5}} \sqrt{3}} \right. \right. \\ \left. \left. - \frac{1}{16} I \sqrt{ -\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 (9 - \sqrt{5} + \sqrt{2} \sqrt{5 + \sqrt{5}} \sqrt{3}) + 4} \right) \right) \left(\frac{1}{3} \right)$ |
| 45 | $\frac{1}{2} \sqrt{2}$ |
| 46 | $\left(-\frac{1}{256} ((\sqrt{5} - 1)(1 + \sqrt{3})\sqrt{5 + \sqrt{5}} - \sqrt{2}(\sqrt{3} - 1)(1 + \sqrt{5})) \sqrt{\sqrt{3}\sqrt{5} - \sqrt{3} + \sqrt{2}\sqrt{5 + \sqrt{5}} + 8} \right. \\ \left. + \frac{1}{256} I \sqrt{ -((\sqrt{5} - 1)(1 + \sqrt{3})\sqrt{5 + \sqrt{5}} - \sqrt{2}(\sqrt{3} - 1)(1 + \sqrt{5}))^2 (\sqrt{3}\sqrt{5} - \sqrt{3} + \sqrt{2}\sqrt{5 + \sqrt{5}} + 8) + 1024} \right) \left(\frac{1}{3} \right) + \left(-\frac{1}{256} ((\sqrt{5} - 1)(1 + \sqrt{3})\sqrt{5 + \sqrt{5}} - \sqrt{2}(\sqrt{3} - 1)(1 + \sqrt{5})) \sqrt{\sqrt{3}\sqrt{5} - \sqrt{3} + \sqrt{2}\sqrt{5 + \sqrt{5}} + 8} \right. \\ \left. - \frac{1}{256} I \sqrt{ -((\sqrt{5} - 1)(1 + \sqrt{3})\sqrt{5 + \sqrt{5}} - \sqrt{2}(\sqrt{3} - 1)(1 + \sqrt{5}))^2 (\sqrt{3}\sqrt{5} - \sqrt{3} + \sqrt{2}\sqrt{5 + \sqrt{5}} + 8) + 1024} \right) \left(\frac{1}{3} \right)$ |
| 47 | $\left(-\frac{1}{256} \sqrt{2} (2\sqrt{3}(1 + \sqrt{5}) + \sqrt{2}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} - \sqrt{6}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} + 2 + 2\sqrt{5}) \right. \\ \left. + \frac{1}{256} I \sqrt{ -2(2\sqrt{3}(1 + \sqrt{5}) + \sqrt{2}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} - \sqrt{6}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} + 2 + 2\sqrt{5})^2 + 1024} \right) \left(\frac{1}{3} \right) + \left(-\frac{1}{256} \sqrt{2} (2\sqrt{3}(1 + \sqrt{5}) + \sqrt{2}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} - \sqrt{6}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} + 2 + 2\sqrt{5}) \right. \\ \left. - \frac{1}{256} I \sqrt{ -2(2\sqrt{3}(1 + \sqrt{5}) + \sqrt{2}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} - \sqrt{6}(\sqrt{5} - 1)\sqrt{5 + \sqrt{5}} + 2 + 2\sqrt{5})^2 + 1024} \right) \left(\frac{1}{3} \right)$ |
| 48 | $\frac{1}{2} \left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right) \sqrt{9 - \sqrt{5} + \sqrt{2} \sqrt{5 + \sqrt{5}} \sqrt{3}}$ |
| 49 | $\left(-\frac{1}{128} \sqrt{2} (\sqrt{5} - 1)(1 + \sqrt{3}) - \frac{1}{64} (\sqrt{3} - 1) \sqrt{5 + \sqrt{5}} + \frac{1}{8} I \sqrt{ -\left(\frac{1}{16} \sqrt{2} (\sqrt{5} - 1)(1 + \sqrt{3}) + \frac{1}{8} (\sqrt{3} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \\ + \left(-\frac{1}{128} \sqrt{2} (\sqrt{5} - 1)(1 + \sqrt{3}) - \frac{1}{64} (\sqrt{3} - 1) \sqrt{5 + \sqrt{5}} - \frac{1}{8} I \sqrt{ -\left(\frac{1}{16} \sqrt{2} (\sqrt{5} - 1)(1 + \sqrt{3}) + \frac{1}{8} (\sqrt{3} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right)$ |
| 50 | $\left(-\frac{1}{16} + \frac{1}{16} I \sqrt{3} \right) \left(\frac{1}{3} \right) + \left(-\frac{1}{16} - \frac{1}{16} I \sqrt{3} \right) \left(\frac{1}{3} \right)$ |
| 51 | $\frac{1}{16} \sqrt{2} \sqrt{3} - \frac{1}{16} \sqrt{5 + \sqrt{5}} \sqrt{3} + \frac{1}{16} \sqrt{5} \sqrt{3} \sqrt{5 + \sqrt{5}} + \frac{1}{16} \sqrt{5} \sqrt{5 + \sqrt{5}} - \frac{1}{16} \sqrt{2} \sqrt{5} - \frac{1}{16} \sqrt{5 + \sqrt{5}} + \frac{1}{16} \sqrt{2} \sqrt{5} \sqrt{3} - \frac{1}{16} \sqrt{2}$ |
| 52 | $\left(-\frac{1}{64} \sqrt{3} (1 + \sqrt{5}) + \frac{1}{128} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} + \frac{1}{8} I \sqrt{ -\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \\ + \left(-\frac{1}{64} \sqrt{3} (1 + \sqrt{5}) + \frac{1}{128} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} - \frac{1}{8} I \sqrt{ -\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right)$ |

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| 53 | $\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right. \\ \left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)+\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}\right. \\ \left.+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 54 | $\frac{1}{4}+\frac{1}{4}\sqrt{5}$ |
| 55 | $\frac{1}{4}2^{\left(\frac{1}{3}\right)}\left(\sqrt{2}(-\sqrt{3}+1+I\sqrt{3}+I)\right)^{\left(\frac{1}{3}\right)}+\frac{1}{4}2^{\left(\frac{1}{3}\right)}\left(-\sqrt{2}(\sqrt{3}-1+I\sqrt{3}+I)\right)^{\left(\frac{1}{3}\right)}$ |
| 56 | $\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right) \\ +\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 57 | $\left(-\frac{1}{64}\sqrt{2}(1+\sqrt{5})+\frac{1}{64}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}(1+\sqrt{5})-\frac{1}{8}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right) \\ +\left(-\frac{1}{64}\sqrt{2}(1+\sqrt{5})+\frac{1}{64}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}(1+\sqrt{5})-\frac{1}{8}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 58 | $\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right) \\ +\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 59 | $\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right. \\ \left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)+\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})\right. \\ \left.+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |
| 60 | $\frac{1}{2}\sqrt{3}$ |
| 61 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right. \\ \left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{384}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})\right. \\ \left.+\frac{1}{192}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right. \\ \left.-\frac{1}{8}I\sqrt{-\left(\frac{1}{48}\sqrt{6}(\sqrt{5}-1)(3+\sqrt{3})-\frac{1}{24}\sqrt{3}(3-\sqrt{3})\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ |

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| 62 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{64}+\frac{1}{64}\sqrt{5}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{6}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}-\frac{1}{8}\sqrt{5}\right)^2+1}\right)\left(\frac{1}{3}\right)$ | |
| 63 | $-\frac{1}{8}\sqrt{2}+\frac{1}{4}\sqrt{5+\sqrt{5}}+\frac{1}{8}\sqrt{2}\sqrt{5}$ | |
| 64 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ $-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{2}\sqrt{5+\sqrt{5}}+\frac{1}{64}\sqrt{3}(\sqrt{5}-1)-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{2}\sqrt{5+\sqrt{5}}-\frac{1}{8}\sqrt{3}(\sqrt{5}-1)\right)^2+1}\right)\left(\frac{1}{3}\right)$ | |
| 65 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1-\frac{1}{3}\sqrt{3}\right)+\frac{1}{32}I\sqrt{-6\left(1-\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{32}\sqrt{6}\left(1-\frac{1}{3}\sqrt{3}\right)-\frac{1}{32}I\sqrt{-6\left(1-\frac{1}{3}\sqrt{3}\right)^2+16}\right)\left(\frac{1}{3}\right)$ | |
| 66 | $\frac{1}{8}+\frac{1}{8}\sqrt{5}-\frac{1}{16}\sqrt{2}\sqrt{3}\sqrt{5+\sqrt{5}}+\frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}$ | |
| 67 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right. \\ \left.+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}+\frac{1}{128}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right. \\ \left.-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}(\sqrt{5}-1)(1+\sqrt{3})\sqrt{5+\sqrt{5}}-\frac{1}{16}\sqrt{2}(\sqrt{3}-1)(1+\sqrt{5})\right)^2+1}\right)\left(\frac{1}{3}\right)$ | |
| 68 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{64}\sqrt{3}(1+\sqrt{5})+\frac{1}{128}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ | |
| 69 | $\frac{1}{2}\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}\sqrt{5}-\frac{1}{16}\sqrt{2}\sqrt{3}\sqrt{5+\sqrt{5}}+\frac{1}{16}\sqrt{2}\sqrt{5}\sqrt{3}\sqrt{5+\sqrt{5}}\right)+\frac{1}{2}\left(\frac{1}{8}\sqrt{3}(1+\sqrt{5})-\frac{1}{16}\sqrt{2}(\sqrt{5}-1)\sqrt{5+\sqrt{5}}\right)\sqrt{2}$ | |
| 70 | $\frac{1}{8}\begin{pmatrix} 2 \\ 3 \end{pmatrix}\left(\begin{pmatrix} 1 \\ 3 \end{pmatrix}\right)\left(\sqrt{2}(-\sqrt{3}-1+I\sqrt{3}-I)\right)+\begin{pmatrix} 1 \\ 3 \end{pmatrix}\left(\begin{pmatrix} 1 \\ 3 \end{pmatrix}\right)\left(\sqrt{2}(-\sqrt{3}+1+I\sqrt{3}-I)\right)$ | |
| 71 | $-\left(-\frac{1}{2}+\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})-\frac{1}{64}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}+\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})+\frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ $-\left(-\frac{1}{2}-\frac{1}{2}I\sqrt{3}\right)\left(-\frac{1}{128}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})-\frac{1}{64}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}-\frac{1}{8}I\sqrt{-\left(\frac{1}{16}\sqrt{2}(\sqrt{5}-1)(1+\sqrt{3})+\frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right)^2+1}\right)\left(\frac{1}{3}\right)$ | |
| 72 | $\frac{1}{4}\sqrt{2}\sqrt{5+\sqrt{5}}$ | $\frac{1}{4}\sqrt{10+2\sqrt{5}}$ |

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| 73 | $\frac{1}{2} \operatorname{sqrt} \left(2 + 2 \left(-\frac{1}{64} \sqrt{2} \sqrt{5 + \sqrt{5}} + \frac{1}{64} \sqrt{3} (\sqrt{5} - 1) \right. \right.$ $\left. \left. + \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{2} \sqrt{5 + \sqrt{5}} - \frac{1}{8} \sqrt{3} (\sqrt{5} - 1) \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right.$ $\left. + 2 \left(-\frac{1}{64} \sqrt{2} \sqrt{5 + \sqrt{5}} + \frac{1}{64} \sqrt{3} (\sqrt{5} - 1) \right. \right.$ $\left. \left. - \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{2} \sqrt{5 + \sqrt{5}} - \frac{1}{8} \sqrt{3} (\sqrt{5} - 1) \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right)$ |
| 74 | $-\left(\frac{1}{2} + \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{256} ((\sqrt{5} - 1)(1 + \sqrt{3}) \sqrt{5 + \sqrt{5}} - \sqrt{2} (\sqrt{3} - 1)(1 + \sqrt{5})) \sqrt{\sqrt{5} \sqrt{3} - \sqrt{3} + \sqrt{2} \sqrt{5 + \sqrt{5}} + 8} \right.$ $\left. + \frac{1}{256} I \sqrt{-((\sqrt{5} - 1)(1 + \sqrt{3}) \sqrt{5 + \sqrt{5}} - \sqrt{2} (\sqrt{3} - 1)(1 + \sqrt{5}))^2 (\sqrt{5} \sqrt{3} - \sqrt{3} + \sqrt{2} \sqrt{5 + \sqrt{5}} + 8) + 1024} \right) \left(\frac{1}{3} \right)$ $-\left(\frac{1}{2} - \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{256} ((\sqrt{5} - 1)(1 + \sqrt{3}) \sqrt{5 + \sqrt{5}} - \sqrt{2} (\sqrt{3} - 1)(1 + \sqrt{5})) \sqrt{\sqrt{5} \sqrt{3} - \sqrt{3} + \sqrt{2} \sqrt{5 + \sqrt{5}} + 8} \right.$ $\left. - \frac{1}{256} I \sqrt{-((\sqrt{5} - 1)(1 + \sqrt{3}) \sqrt{5 + \sqrt{5}} - \sqrt{2} (\sqrt{3} - 1)(1 + \sqrt{5}))^2 (\sqrt{5} \sqrt{3} - \sqrt{3} + \sqrt{2} \sqrt{5 + \sqrt{5}} + 8) + 1024} \right) \left(\frac{1}{3} \right)$ |
| 75 | $\frac{1}{4} \sqrt{2} (1 + \sqrt{3})$ |
| 76 | $\frac{1}{2} \operatorname{sqrt} \left(2 - 2 \left(\frac{1}{2} + \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{128} \sqrt{6} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} + \frac{1}{64} + \frac{1}{64} \sqrt{5} + \frac{1}{8} I \sqrt{-\left(\frac{1}{16} \sqrt{6} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} - \frac{1}{8} - \frac{1}{8} \sqrt{5} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right.$ $\left. - 2 \left(\frac{1}{2} - \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{128} \sqrt{6} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} + \frac{1}{64} + \frac{1}{64} \sqrt{5} - \frac{1}{8} I \sqrt{-\left(\frac{1}{16} \sqrt{6} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} - \frac{1}{8} - \frac{1}{8} \sqrt{5} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right)$ |
| 77 | $\frac{1}{2} \operatorname{sqrt} \left(2 - 2 \left(\frac{1}{2} + \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{64} \sqrt{2} \sqrt{5 + \sqrt{5}} + \frac{1}{64} \sqrt{3} (\sqrt{5} - 1) + \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{2} \sqrt{5 + \sqrt{5}} - \frac{1}{8} \sqrt{3} (\sqrt{5} - 1) \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right.$ $\left. - 2 \left(\frac{1}{2} - \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{64} \sqrt{2} \sqrt{5 + \sqrt{5}} + \frac{1}{64} \sqrt{3} (\sqrt{5} - 1) - \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{2} \sqrt{5 + \sqrt{5}} - \frac{1}{8} \sqrt{3} (\sqrt{5} - 1) \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right)$ |
| 78 | $-\frac{1}{8} + \frac{1}{8} \sqrt{5} + \frac{1}{8} \sqrt{2} \sqrt{3} \sqrt{5 + \sqrt{5}}$ |
| 79 | $\frac{1}{2} \operatorname{sqrt} \left(2 - 2 \left(\frac{1}{2} + \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{64} \sqrt{3} (1 + \sqrt{5}) + \frac{1}{128} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} + \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right.$ $\left. - 2 \left(\frac{1}{2} - \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{64} \sqrt{3} (1 + \sqrt{5}) + \frac{1}{128} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} - \frac{1}{8} I \sqrt{-\left(\frac{1}{8} \sqrt{3} (1 + \sqrt{5}) - \frac{1}{16} \sqrt{2} (\sqrt{5} - 1) \sqrt{5 + \sqrt{5}} \right)^2 + 1} \right) \left(\frac{1}{3} \right) \right)$ |
| 80 | $-\left(\frac{1}{2} + \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{16} \sqrt{3} + \frac{1}{16} I \right) \left(\frac{1}{3} \right) - \left(\frac{1}{2} - \frac{1}{2} I \sqrt{3} \right) \left(-\frac{1}{16} \sqrt{3} - \frac{1}{16} I \right) \left(\frac{1}{3} \right)$ |
| 81 | $-\frac{1}{8} \sqrt{5 + \sqrt{5}} + \frac{1}{8} \sqrt{5} \sqrt{5 + \sqrt{5}} + \frac{1}{8} \sqrt{2} \sqrt{5} + \frac{1}{8} \sqrt{2}$ |

