Somewhat Random Numbers

Prepared by Mark on February 14, 2025

Instructor's Handout

This handout contains solutions and notes.
Recompile without solutions before distributing.

Problem 1:
Alice generates 100 random numbers uniformly from $[0, 1]$.
Bob generates 101 random numbers from $[0, 1]$, but deletes the lowest result.

Say we have both of the resulting arrays, but do not know who generated each one.
We would like to guess which of the two was generated by Bob.

- Say we assign the array with the smallest mean to Alice.
  What is our probability of guessing correctly?
- How often do we guess correctly if we instead assign
  the array with the smaller *minimum* to Alice?

Solution:

Looking at the mean seems like a good idea, but there's a better way:
Assign the array with the smaller *minimum* to Alice.

To compute the probability, generate 201 numbers.
Assign the first 100 to Alice and the rest to Bob.
Look at the lowest two numbers (of these 201, **before** Bob drops his lowest).

We'll use the following notation:
`AB` means the lowest was owned by Alice, and the second-lowest, by Bob.

Probabilities are as follows:

- `AA`: $100/201 \times 99/200 \approx 0.246$
- `AB`: $100/201 \times 101/200 \approx 0.251$
- `BA`: $101/201 \times 100/200 \approx 0.251$
- `BB`: $101/201 \times 100/200 \approx 0.251$

Now, Bob drops his lowest number.
We'll cross out the number he drops and box the new lowest number (i.e, the one we observe):

- $\boxed{\texttt{A}}\texttt{A}$: ≈ 0.246
- $\boxed{\texttt{A}}\cancel{\texttt{B}}$: ≈ 0.251
- $\cancel{\texttt{B}}\boxed{\texttt{A}}$: ≈ 0.251
- $\cancel{\texttt{B}}\boxed{\texttt{B}}$: ≈ 0.251

Alice has the smallest number in 3 of 4 cases, which have a total probability of ≈ 0.749.