Warm-Up: Mario Kart

Prepared by Mark on January 25, 2025

Instructor's Handout

This handout contains solutions and notes.
Recompile without solutions before distributing.

Problem 1:
A standard Mario Kart cup consists of 12 players and four races.
Each race is scored as follows:

- 15 points are awarded for first place;
- 12 for second;
- and $(13 - \text{place})$ otherwise.

In any one race, no players may tie.
A player's score at the end of a cup is the sum of their scores for each of the four races.

An n-way tie occurs when the top n players have the same score at the end of a round.
What is the largest possible n, and how is it achieved?

Solution:

A 12-way tie is impossible, since the total number of point is not divisible by 12.

A 11-way tie is possible, with a top score of 28:

- Four players finish 1^{st}, 3^{ed}, 11^{th}, and 12^{th};
- Four players finish 2^{nd}, 4^{th}, 9^{th}, and 10^{th};
- Two players finish fifth twice and seventh twice,
- One player finishes sixth in each race.

The final player always finishes eighth, with a non-tie score of 20.