

# Pidgeonhole Problems

Prepared by Mark on January 25, 2025

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**Problem 1:**

Is it possible to cover an equilateral triangle with two smaller equilateral triangles? Why or why not?

**Problem 2:**

You are given  $n + 1$  distinct integers.

Prove that at least two of them have a difference divisible by  $n$ .

**Problem 3:**

You have an  $8 \times 8$  chess board with two opposing corner squares cut off. You also have a set of dominoes, each of which is the size of two squares. Is it possible to completely cover the board with dominos, so that none overlap nor stick out?

**Problem 4:**

The ocean covers more than a half of the Earth's surface. Prove that the ocean has at least one pair of antipodal points.

**Problem 5:**

There are  $n > 1$  people at a party. Prove that among them there are at least two people who have the same number of acquaintances at the gathering. (We assume that if A knows B, then B also knows A)

**Problem 6:**

Pick five points in  $\mathbb{R}^2$  with integral coordinates. Show that two of these form a line segment that has an integral midpoint.

**Problem 7:**

Every point on a line is painted black or white. Show that there exist three points of the same color where one is the midpoint of the line segment formed by the other two.

**Problem 8:**

Every point on a plane is painted black or white. Show that there exist two points in the plane that have the same color and are located exactly one foot away from each other.

**Problem 9:**

Let  $n$  be an integer not divisible by 2 and 5. Show that  $n$  has a multiple consisting entirely of ones.

**Problem 10:**

Prove that for any  $n > 1$ , there exists an integer made of only sevens and zeros that is divisible by  $n$ .

**Problem 11:**

Choose  $n + 1$  integers between 1 and  $2n$ . Show that at least two of these are co-prime.

**Problem 12:**

Choose  $n + 1$  integers between 1 and  $2n$ . Show that you must select two numbers  $a$  and  $b$  such that  $a$  divides  $b$ .

**Problem 13:**

Show that it is always possible to choose a subset of the set of integers  $\{a_1, a_2, \dots, a_n\}$  so that the sum of the numbers in the subset is divisible by  $n$ .

**Problem 14:**

Show that there exists a positive integer divisible by 2013 that has 2014 as its last four digits.

**Problem 15:**

Let  $n$  be an odd number. Let  $a_1, a_2, \dots, a_n$  be a permutation of the numbers  $1, 2, \dots, n$ . Show that  $(a_1 - 1) \times (a_2 - 2) \times \dots \times (a_n - n)$  is even.

**Problem 16:**

A stressed-out student consumes at least one espresso every day of a particular year, drinking 500 overall. Show the student drinks exactly 100 espressos on some consecutive sequence of days.

**Problem 17:**

Show that there are either three mutual acquaintances or four mutual strangers at a party with ten or more people.

**Problem 18:**

Given a table with a marked point,  $O$ , and with 2013 properly working watches put down on the table, prove that there exists a moment in time when the sum of the distances from  $O$  to the watches' centers is less than the sum of the distances from  $O$  to the tips of the watches' minute hands.