

Slide Rules

Prepared by Mark on January 23, 2025

Dad says that anyone who can't use
a slide rule is a cultural illiterate and
should not be allowed to vote.

Have Space Suit — Will Travel, 1958

Part 1: Logarithms

Definition 1:

The *logarithm* is the inverse of the exponent. That is, if $b^p = c$, then $\log_b c = p$.

In other words, $\log_b c$ asks the question “what power do I need to raise b to to get c ?”

In both b^p and $\log_b c$, the number b is called the *base*.

Problem 1:

Evaluate the following by hand:

A: $\log_{10}(1000)$

B: $\log_2(64)$

C: $\log_2\left(\frac{1}{4}\right)$

D: $\log_x(x)$ for any x

E: $\log_x(1)$ for any x

Definition 2:

There are a few ways to write logarithms:

$$\log x = \log_{10} x$$

$$\lg x = \log_{10} x$$

$$\ln x = \log_e x$$

Definition 3:

The *domain* of a function is the set of values it can take as inputs.

The *range* of a function is the set of values it can produce.

For example, the domain and range of $f(x) = x$ is $\mathbb{R}$, all real numbers.

The domain of $f(x) = |x|$ is $\mathbb{R}$, and its range is $\mathbb{R}^+ \cup \{0\}$, all positive real numbers and 0.

Note that the domain and range of a function are not always equal.

Problem 2:

What is the domain of $f(x) = 5^x$?

What is the range of $f(x) = 5^x$?

Problem 3:

What is the domain of $f(x) = \log x$?

What is the range of $f(x) = \log x$?

Problem 4:

Prove the following identities:

A: $\log_b(b^x) = x$

B: $b^{\log_b x} = x$

C: $\log_b(xy) = \log_b(x) + \log_b(y)$

D: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

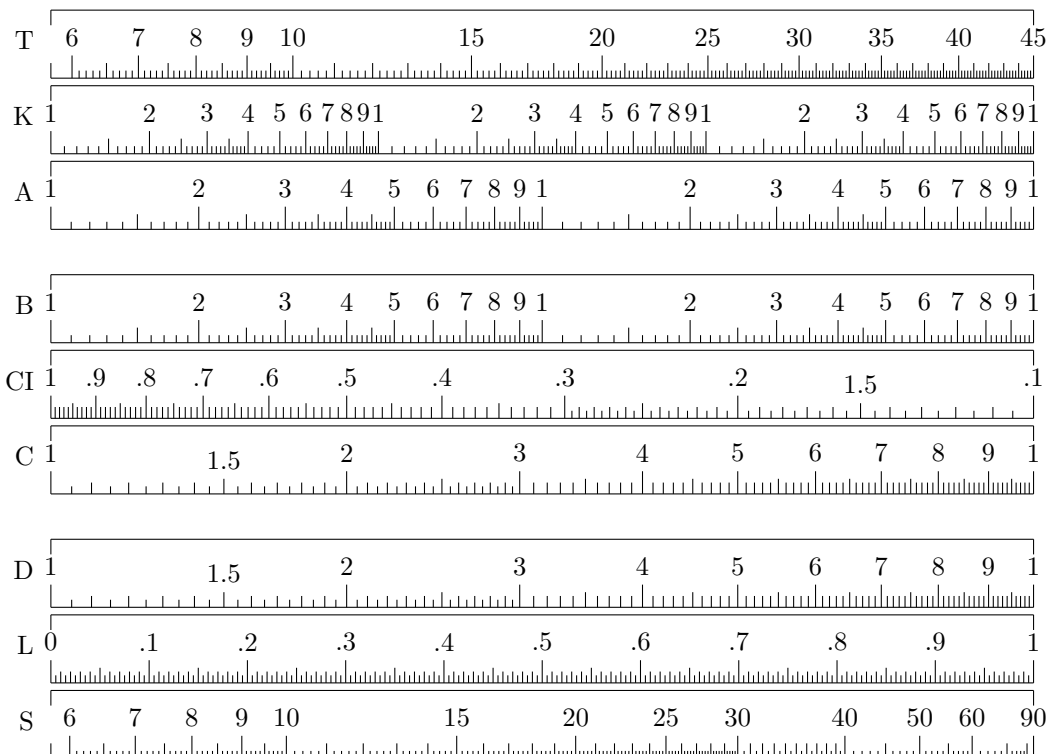
E: $\log_b(x^y) = y \log_b(x)$

Part 2: Introduction

Mathematicians, physicists, and engineers needed to quickly solve complex equations even before computers were invented.

The *slide rule* is an instrument that uses the logarithm to solve this problem. Before you continue, cut out and assemble your slide rule.

There are four scales on your slide rule, each labeled with a letter on the left side:



Each scale's "generating function" is on the right:

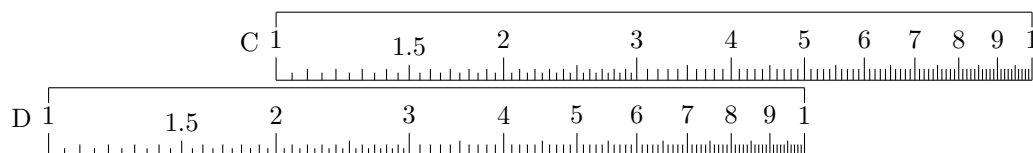
- T: $\tan$
- K: x^3
- A, B: x^2
- CI: $\frac{1}{x}$
- C, D: x
- L: $\log_{10}(x)$
- S: $\sin$

Once you understand the layout of your slide rule, move on to the next page.

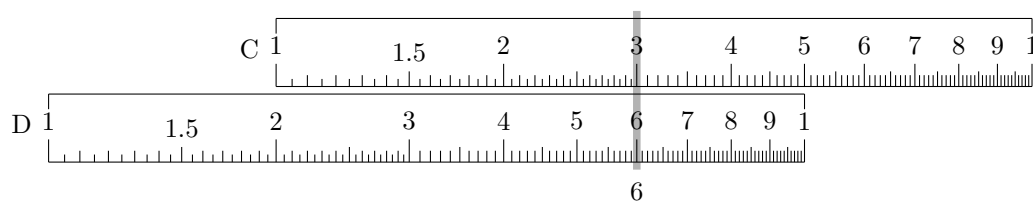
Part 3: Multiplication

We'll use the C and D scales of your slide rule to multiply.

Say we want to multiply 2×3 . First, move the *left-hand index* of the C scale over the smaller number, 2:



Then we'll find the second number, 3 on the C scale, and read the D scale under it:



Of course, our answer is 6.

Problem 5:

What is 1.15×2.1 ?

Use your slide rule.

Note that your answer isn't exact. $1.15 \times 2.1 = 2.415$, but an answer accurate within two decimal places is close enough for most practical applications.

Look at your C and D scales again. They contain every number between 1 and 10, but no more than that. What should we do if we want to calculate 32×210 ?

Problem 6:

Using your slide rule, calculate 32×210 .

Problem 7:

Compute the following:

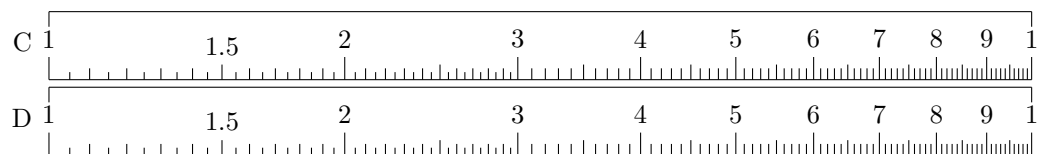
A: 1.44×52

B: 0.38×1.24

C: $\pi \times 2.35$

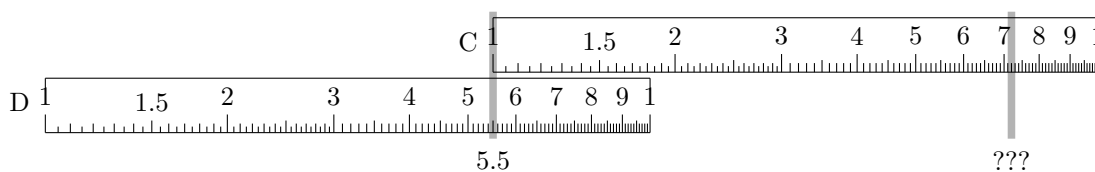
Problem 8:

Note that the numbers on your C and D scales are logarithmically spaced.



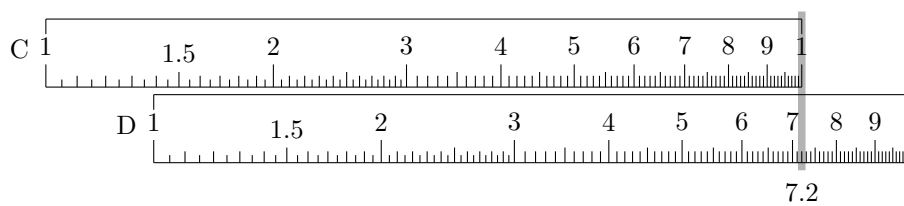
Why does our multiplication procedure work?

Now we want to compute 7.2×5.5 :

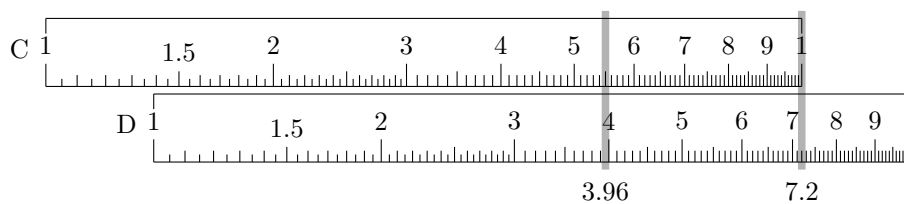


No matter what order we go in, the answer ends up off the scale. There must be another way.

Look at the far right of your C scale. There's an arrow pointing to the 10 tick, labeled *right-hand index*. Move it over the *larger* number, 7.2:



Now find the smaller number, 5.5, on the C scale, and read the D scale under it:



Our answer should be about $7 \times 5 = 35$, so let's move the decimal point: $5.5 \times 7.2 = 39.6$. We can do this by hand to verify our answer.

Problem 9:

Why does this work?

Problem 10:

Compute the following using your slide rule:

A: 9×8

B: 15×35

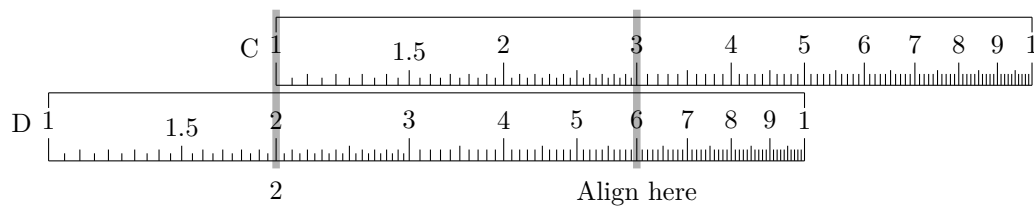
C: 42.1×7.65

D: 6.5^2

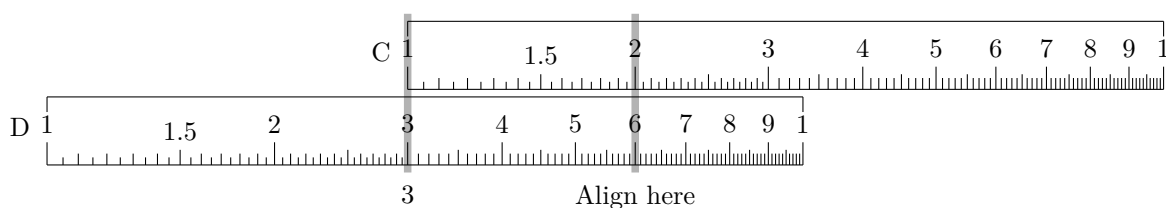
Part 4: Division

Now that you can multiply, division should be easy. All you need to do is work backwards. Let's look at our first example again: $3 \times 2 = 6$.

We can easily see that $6 \div 3 = 2$

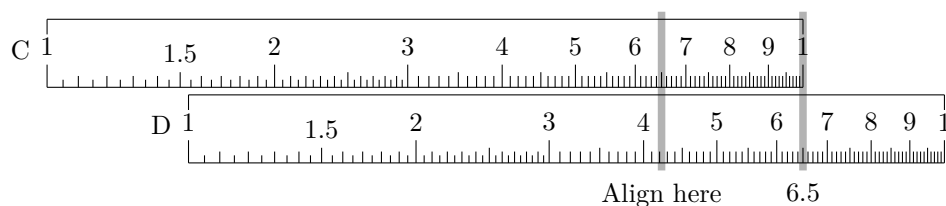


and that $6 \div 2 = 3$:



If your left-hand index is off the scale, read the right-hand one.

Consider $42.25 \div 6.5 = 6.5$:



Place your decimal points carefully.

Problem 11:

Compute the following using your slide rule.

A: $135 \div 15$

B: $68.2 \div 0.575$

C: $(118 \times 0.51) \div 6.6$

Part 5: Squares, Cubes, and Roots

Now, take a look at scales A and B, and note the label on the right: x^2 . If C, D are x , A and B are x^2 , and K is x^3 .

Finding squares of numbers up to ten is straightforward: just read the scale.

Square roots are also easy: find your number on B and read its pair on C.



Problem 12:

Compute the following.

A: 1.5^2

B: 3.1^2

C: 7^3

D: $\sqrt{14}$

E: $\sqrt[3]{150}$

Problem 13:

Compute the following.

A: 42^2

B: $\sqrt{200}$

C: $\sqrt{2000}$

D: $\sqrt{0.9}$

E: $\sqrt[3]{0.12}$

Part 6: Inverses

Try finding $1 \div 32$ using your slide rule.

The procedure we learned before doesn't work!

This is why we have the CI scale, or the “C Inverse” scale.

Problem 14:

Figure out how the CI scale works and compute the following:

A: $\frac{1}{7}$

B: $\frac{1}{120}$

C: $\frac{1}{\pi}$

Part 7: Logarithms Base 10

When we take a logarithm, the resulting number has two parts: the *characteristic* and the *mantissa*. The characteristic is the integral (whole-numbered) part of the answer, and the mantissa is the fractional part (what comes after the decimal).

For example, $\log_{10} 18 = 1.255$, so in this case the characteristic is 1 and the mantissa is 0.255.

Problem 15:

Approximate the following logs without a slide rule. Find the exact characteristic, and approximate the mantissa.

A: $\log_{10} 20$

B: $\log_2 18$

Now, find the L scale on your slide rule. As you can see on the right, its generating function is $\log_{10} x$.

Problem 16:

Compute the following logarithms using your slide rule.

You'll have to find the characteristic yourself, but your L scale will give you the mantissa.

Don't forget your log identities!

A: $\log_{10} 20$

B: $\log_{10} 15$

C: $\log_{10} 150$

D: $\log_{10} 0.024$

Part 8: Logarithms in Any Base

Our slide rule easily computes logarithms in base 10, but we can also use it to find logarithms in *any* base.

Proposition 1:

This is usually called the *change-of-base* formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Problem 17:

Using log identities, prove ??.

Problem 18:

Approximate the following:

- A:** $\log_2 56$
- B:** $\log_{5.2} 26$
- C:** $\log_{12} 500$
- D:** $\log_{43} 134$

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