To Mock a Mockingbird

Prepared by Mark on February 15, 2026
Based on a book of the same name.

Instructor's Handout

This file contains solutions and notes.
Compile with the "nosolutions" flag before distributing.
Click `[here]` for the latest version of this handout.

Part 1: Introduction

A certain enchanted forest is inhabited by talking birds. Each of these birds has a name, and will respond whenever it hears the name of another. Suppose you are exploring this forest and come across the bird A. You call the name of bird B. A hears you and responds with the name of some other bird, which we will designate AB.
Bird AB is, by definition, A's response to B.

As you wander around this forest, you quickly discover two interesting facts:

A: A's response to B mustn't be the same as B's response to A.

B: Given three birds A, B, and C, $(AB)C$ and $A(BC)$ are not necessarily the same bird.
Bird $A(BC)$ is A's response to bird BC, while $(AB)C$ is AB's response to C.
Thus, ABC is ambiguous. Parenthesis are mandatory.

You also find that this forest has two laws:

A: *The Law of Composition*:
For any two birds A and B, there must be a bird C so that $Cx = A(Bx)$

B: *The Law of the Mockingbird*:
The forest must contain the Mockingbird M, which always satisfies $Mx = xx$.
In other words, the Mockingbird's response to any bird x is the same as x's response to itself.

Definition 1:
We say a bird A is fond of a bird B if A responds to B with B.
In other words, A is fond of B if $AB = B$.

Definition 2:
We say a bird C *composes* A with B if for any bird x,

$$Cx = A(Bx)$$

In other words, this means that C's response to x is the same as A's response to B's response to x.
Note that C is exactly the kind of bird L_1 guarantees.

Part 2: To Mock a Mockingbird

Problem 3:
Mark tells you that any bird A is fond of at least one other bird.
Complete his proof.

```
let A           # Let A be any any bird.
let Cx = A(Mx)  # Define C as the composition of A and M

# The rest is up to you.
CC = ??
```

Things you will need:

`Law:` There exists a Mockingbird, $Mx := xx$
`Def:` A is fond of B if $AB = B$

Solution

```
01 let A           # Let A be any any bird.
02 let Cx = A(Mx)  # Define C as the composition of A and M
03 CC = A(MC)
04    = A(CC)
```

Problem 4:
We say a bird A is *egocentric* if it is fond of itself.
Show that the laws of the forest guarantee that at least one bird is egocentric.

Things you will need:

`Law:` There exists a Mockingbird, $Mx := xx$
`Def:` A is fond of B if $AB = B$
`Lem:` Any bird is fond of at least one bird.

Solution

```
01 # We know M is fond of at least one bird.
02 let E so that ME = E
03
04 ME = E     # By definition of fondness
05 ME = EE    # By definition of M
06 ⇒ EE = E
```

Definition 5:
We say a bird A is *agreeable* if for all birds B, there is at least one bird x on which A and B agree. In other words, A is agreeable if given any B, we can find a bird x satisfying $Ax = Bx$.

Problem 6:
Is the Mockingbird agreeable?

Solution

We know that $Mx = xx$.
From this definition, we see that M agrees with any x on x itself.

Problem 7:
Take two birds A and B. Let C be their composition.
Show that if C is agreeable, A is agreeable.

```
# Given information
let A, B
let Cx = A(Bx)

let D            # Arbitrary bird
let Ex = D(Bx)   # Define E as the composition of D and B
Cy = ??
```

Things you will need:

`Def:` A is agreeable if $Ax = Bx$ for all B with some x.
`Law:` For any A, B, there is C defined by Cx = A(Bx)

Solution

```
01 # Given information
02 let A, B
03 let Cx = A(Bx)
04
05 let D                  # Arbitrary bird
06 let Ex = D(Bx)         # Define E as the composition of D and B
07 let y so that Cy = Ey  # Such a y must exist because C is agreeable
08
09 A(By) = Ey
10       = D(By)
```

Problem 8:
Given three arbitrary birds A, B, and C, show that there exists a bird D satisfying $Dx = A(B(Cx))$

Solution

```
01 let A, B, C
02
03 # Invoke the Law of Composition:
04 let Qx = B(Cx)
05 let Dx = A(Qx)
06
07 Dx = A(Qx)
08   = A(B(Cx))  ■
```

Definition 9:
We say two birds A and B are *compatible* if there are birds x and y so that $Ax = y$ and $By = x$. Note that x and y may be the same bird.

Problem 10:
Show that any bird that is fond of at least one bird is compatible with itself.

Solution

```
01 let A
02 let x so that Ax = x  # A is fond of at least one other bird
03 Ax = x
```

Problem 11:
Show that any two birds in this forest are compatible.

```
let A, B
let Cx = A(Bx)
```

Things you will need:

`Law:` Law of composition
`Lem:` Any bird is fond of at least one bird.

Solution

```
01 let A, B
02 let Cx = A(Bx)  # Composition
03 let y = Cy      # Let C be fond of y
04
05 Cy = y
06    = A(By)
07
08 let x = By  # Rename By to x
09 Ax = y
```

Part 3: The Curious Kestrel

Definition 12:
Recall that a bird is *egocentric* if it is fond of itself.
A bird is *hopelessly egocentric* if $Bx = B$ for all birds x.

Definition 13:
More generally, we say that a bird A is *fixated* on a bird B if $Ax = B$ for all x.
Convince yourself that a hopelessly egocentric bird is fixated on itself.

Problem 14:
Say A is fixated on B. Is A fond of B?

Solution

Yes! See the following proof.

```
01 let A
02 let B so that Ax = B
03 ⇒ AB = B
```

Definition 15:
The *Kestrel* K is defined by the following relationship:

$$(Kx)y = x \quad \forall x, y$$

In other words, this means that for every bird x, the bird Kx is fixated on x.

Problem 16:
Show that an egocentric Kestrel is hopelessly egocentric.

Solution

```
01 KK = K
02 ⇒ (KK)y = K  # By definition of the Kestrel
03 ⇒ Ky = K     # By 01
```

Problem 17:
Assume the forest contains a Kestrel.
Given the Law of Composition and the Law of the Mockingbird, show that at least one bird is hopelessly egocentric.

Things you will need:

`Def:` K is defined by $(Kx)y = x$
`Def:` A is fond of B if $AB = B$
`???:` You'll need one more result from the previous section. Good luck!

Solution

The final piece is a lemma we proved earlier:
Any bird is fond of at least one bird

```
01 let A so that KA = A   # Any bird is fond of at least one bird
02 (KA)y = y              # By definition of the kestrel
03 ⇒ Ay = A               # By 01
```

Problem 18: Kestrel Left-Cancellation
In general, $Ax = Ay$ does not imply $x = y$. However, this is true if A is K.
Show that $Kx = Ky \implies x = y$.

```
# This is a hint.
let x, y so that Kx = Ky
```

Solution

```
01 let x, y so that Kx = Ky
02 let z
03
04 (Kx)z = (Ky)z  # By 01
05
06 # By the definition of K
07 (Kx)z = x
08 (Ky)z = y
09
10 ⇒ x = (Kx)z = (Ky)z = y
```

Problem 19:
Show that if K is fond of Kx, K is fond of x.

Solution

```
01 let x so that K(Kx) = Kx
02 (K(Kx))y = (Kx)y
03          = Kx  # By definition of K
04 x = Kx  # By 03 and definition of K
```

Problem 20:
An egocentric Kestrel must be extremely lonely. Why is this?

Solution

If a Kestrel is egocentric, it must be the only bird in the forest!

```
01 # Given
02 Kx = K for some x
03 # We have shown that an egocentric kestrel is hopelessly egocentric
04 Kx = K for all x
05
06 let x, y
07 Kx = K
08 Ky = K
09 Kx = Ky
10 x = y for all x, y  # By Problem 18
11 x = y = K  # By 10, and since K exists
```

Part 4: Bonus Problems

Definition 21:
The identity bird has sometimes been maligned, owing to the fact that whatever bird x you call to I, all I does is to echo x back to you.

Superficially, the bird I appears to have no intelligence or imagination; all it can do is repeat what it hears. For this reason, in the past, thoughtless students of ornithology referred to it as the idiot bird. However, a more profound or- nithologist once studied the situation in great depth and dis- covered that the identity bird is in fact highly intelligent! The real reason for its apparently unimaginative behavior is that it has an unusually large heart and hence is fond of every bird! When you call x to I, the reason it responds by calling back x is not that it can't think of anything else; it's just that it wants you to know that it is fond of x!

Since an identity bird is fond of every bird, then it is also fond of itself, so every identity bird is egocentric. However, its egocentricity doesn't mean that it is any more fond of itself than of any other bird!.

Problem 22:
The laws of the forest no longer apply.
Suppose we are told that the forest contains an identity bird I and that I is agreeable. Does it follow that every bird must be fond of at least one bird?

Problem 23:
Suppose we are told that there is an identity bird I and that every bird is fond of at least one bird. Does it necessarily follow that I is agreeable?

Problem 24:
Suppose we are told that there is an identity bird I, but we are not told whether I is agreeable or not. However, we are told that every pair of birds is compatible. Which of the following conclusiens can be validly drawn?

- Every bird is fond of at least one bird
- I is agreeable.

Problem 25:
The identity bird I, though egocentric, is in general not hope- lessly egocentric. Indeed, if there were a hopelessly egocentric identity bird, the situation would be quite sad. Why?

Definition 26:
A bird L is called a lark if the following holds for any birds x and y:

$$(Lx)y = x(yy)$$

Problem 27:
Prove that if the forest contains a lark L and an identity bird I, then it must also contain a mockingbird M.

Problem 28:
Why is a hopelessly egocentric lark unusually attractive?

Problem 29:
Assuming that no bird can be both a lark and a kestrel—as any ornithologist knows!—prove that it is impossible for a lark to be fond of a kestrel.

Problem 30:
It might happen, however, that a kestrel is fond of a lark.
Show that in this case, *every* bird is fond of the lark.