

# Slide Rules

Prepared by Mark on January 27, 2025

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Dad says that anyone who can't use  
a slide rule is a cultural illiterate and  
should not be allowed to vote.

*Have Space Suit — Will Travel, 1958*

## Part 1: Logarithms

### Definition 1:

The *logarithm* is the inverse of the exponent. That is, if  $b^p = c$ , then  $\log_b c = p$ .

In other words,  $\log_b c$  asks the question “what power do I need to raise  $b$  to to get  $c$ ?”

In both  $b^p$  and  $\log_b c$ , the number  $b$  is called the *base*.

### Problem 1:

Evaluate the following by hand:

**A:**  $\log_{10}(1000)$

**B:**  $\log_2(64)$

**C:**  $\log_2\left(\frac{1}{4}\right)$

**D:**  $\log_x(x)$  for any  $x$

**E:**  $\log_x(1)$  for any  $x$

**Definition 2:**

There are a few ways to write logarithms:

$$\log x = \log_{10} x$$

$$\lg x = \log_{10} x$$

$$\ln x = \log_e x$$

**Definition 3:**

The *domain* of a function is the set of values it can take as inputs.

The *range* of a function is the set of values it can produce.

For example, the domain and range of  $f(x) = x$  is  $\mathbb{R}$ , all real numbers.

The domain of  $f(x) = |x|$  is  $\mathbb{R}$ , and its range is  $\mathbb{R}^+ \cup \{0\}$ , all positive real numbers and 0.

Note that the domain and range of a function are not always equal.

**Problem 2:**

What is the domain of  $f(x) = 5^x$ ?

What is the range of  $f(x) = 5^x$ ?

**Problem 3:**

What is the domain of  $f(x) = \log x$ ?

What is the range of  $f(x) = \log x$ ?

**Problem 4:**

Prove the following identities:

**A:**  $\log_b(b^x) = x$

**B:**  $b^{\log_b x} = x$

**C:**  $\log_b(xy) = \log_b(x) + \log_b(y)$

**D:**  $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

**E:**  $\log_b(x^y) = y \log_b(x)$

## Part 2: Introduction

Mathematicians, physicists, and engineers needed to quickly solve complex equations even before computers were invented.

The *slide rule* is an instrument that uses the logarithm to solve this problem. Before you continue, cut out and assemble your slide rule.

There are four scales on your slide rule, each labeled with a letter on the left side:



Each scale's “generating function” is on the right:

- T:  $\tan$
- K:  $x^3$
- A, B:  $x^2$
- CI:  $\frac{1}{x}$
- C, D:  $x$
- L:  $\log_{10}(x)$
- S:  $\sin$

Once you understand the layout of your slide rule, move on to the next page.

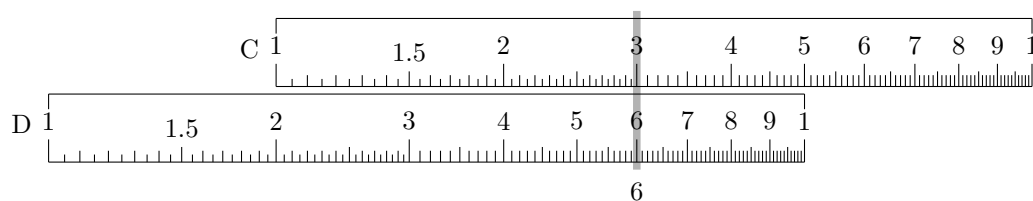
## Part 3: Multiplication

We'll use the C and D scales of your slide rule to multiply.

Say we want to multiply  $2 \times 3$ . First, move the *left-hand index* of the C scale over the smaller number, 2:



Then we'll find the second number, 3 on the C scale, and read the D scale under it:



Of course, our answer is 6.

### Problem 5:

What is  $1.15 \times 2.1$ ?

Use your slide rule.

Note that your answer isn't exact.  $1.15 \times 2.1 = 2.415$ , but an answer accurate within two decimal places is close enough for most practical applications.

Look at your C and D scales again. They contain every number between 1 and 10, but no more than that. What should we do if we want to calculate  $32 \times 210$ ?

**Problem 6:**

Using your slide rule, calculate  $32 \times 210$ .

**Problem 7:**

Compute the following:

**A:**  $1.44 \times 52$

**B:**  $0.38 \times 1.24$

**C:**  $\pi \times 2.35$

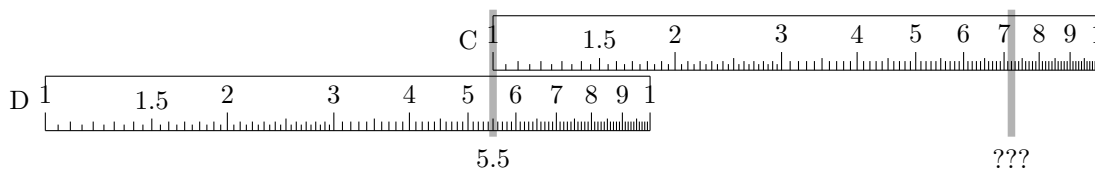
**Problem 8:**

Note that the numbers on your C and D scales are logarithmically spaced.



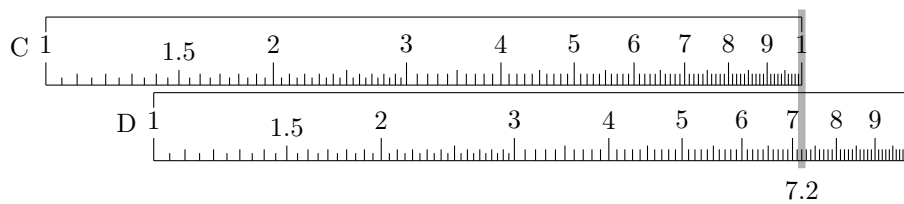
Why does our multiplication procedure work?

Now we want to compute  $7.2 \times 5.5$ :

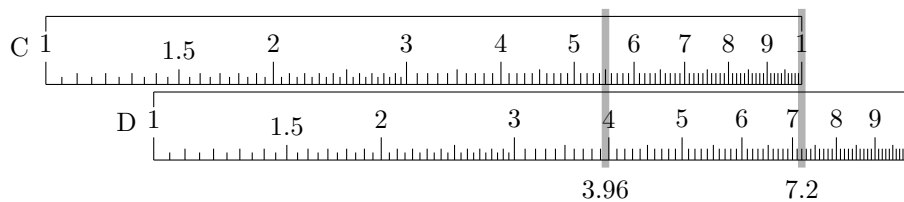


No matter what order we go in, the answer ends up off the scale. There must be another way.

Look at the far right of your C scale. There's an arrow pointing to the 10 tick, labeled *right-hand index*. Move it over the *larger* number, 7.2:



Now find the smaller number, 5.5, on the C scale, and read the D scale under it:



Our answer should be about  $7 \times 5 = 35$ , so let's move the decimal point:  $5.5 \times 7.2 = 39.6$ . We can do this by hand to verify our answer.

### Problem 9:

Why does this work?



**Problem 10:**

Compute the following using your slide rule:

**A:**  $9 \times 8$

**B:**  $15 \times 35$

**C:**  $42.1 \times 7.65$

**D:**  $6.5^2$

## Part 4: Division

Now that you can multiply, division should be easy. All you need to do is work backwards. Let's look at our first example again:  $3 \times 2 = 6$ .

We can easily see that  $6 \div 3 = 2$

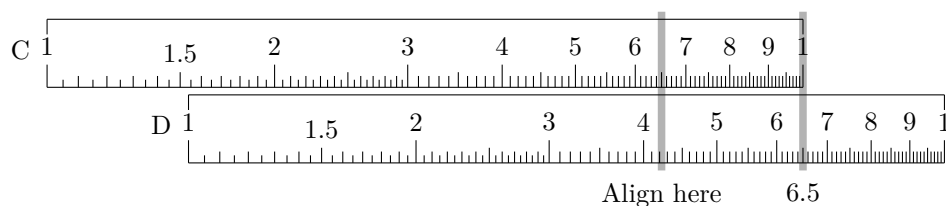


and that  $6 \div 2 = 3$ :



If your left-hand index is off the scale, read the right-hand one.

Consider  $42.25 \div 6.5 = 6.5$ :



Place your decimal points carefully.

**Problem 11:**

Compute the following using your slide rule.

**A:**  $135 \div 15$

**B:**  $68.2 \div 0.575$

**C:**  $(118 \times 0.51) \div 6.6$

## Part 5: Squares, Cubes, and Roots

Now, take a look at scales A and B, and note the label on the right:  $x^2$ . If C, D are  $x$ , A and B are  $x^2$ , and K is  $x^3$ .

Finding squares of numbers up to ten is straightforward: just read the scale.

Square roots are also easy: find your number on B and read its pair on C.



### Problem 12:

Compute the following.

**A:**  $1.5^2$

**B:**  $3.1^2$

**C:**  $7^3$

**D:**  $\sqrt{14}$

**E:**  $\sqrt[3]{150}$

### Problem 13:

Compute the following.

**A:**  $42^2$

**B:**  $\sqrt{200}$

**C:**  $\sqrt{2000}$

**D:**  $\sqrt{0.9}$

**E:**  $\sqrt[3]{0.12}$

## Part 6: Inverses

Try finding  $1 \div 32$  using your slide rule.

The procedure we learned before doesn't work!

This is why we have the CI scale, or the “C Inverse” scale.

### Problem 14:

Figure out how the CI scale works and compute the following:

**A:**  $\frac{1}{7}$

**B:**  $\frac{1}{120}$

**C:**  $\frac{1}{\pi}$

## Part 7: Logarithms Base 10

When we take a logarithm, the resulting number has two parts: the *characteristic* and the *mantissa*. The characteristic is the integral (whole-numbered) part of the answer, and the mantissa is the fractional part (what comes after the decimal).

For example,  $\log_{10} 18 = 1.255$ , so in this case the characteristic is 1 and the mantissa is 0.255.

### Problem 15:

Approximate the following logs without a slide rule. Find the exact characteristic, and approximate the mantissa.

**A:**  $\log_{10} 20$

**B:**  $\log_2 18$

Now, find the L scale on your slide rule. As you can see on the right, its generating function is  $\log_{10} x$ .

### Problem 16:

Compute the following logarithms using your slide rule.

You'll have to find the characteristic yourself, but your L scale will give you the mantissa.

Don't forget your log identities!

**A:**  $\log_{10} 20$

**B:**  $\log_{10} 15$

**C:**  $\log_{10} 150$

**D:**  $\log_{10} 0.024$

## Part 8: Logarithms in Any Base

Our slide rule easily computes logarithms in base 10, but we can also use it to find logarithms in *any* base.

**Proposition 1:**

This is usually called the *change-of-base* formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

**Problem 17:**

Using log identities, prove ??.

**Problem 18:**

Approximate the following:

- A:**  $\log_2 56$
- B:**  $\log_{5.2} 26$
- C:**  $\log_{12} 500$
- D:**  $\log_{43} 134$

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