

Warm-Up: Passing Balls

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Instructor's Handout

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Problem 1:

Twelve people are standing in a circle. Each is assigned a number between 1 and 12. Participants numbered 1, 2, 3, and 4 hold red, green, yellow, and black balls, respectively. Everyone else is empty-handed.

Each participant can pass their ball to any student that is exactly 5 positions away. Balls cannot be passed to someone who has one in hand.

After a number of passes, the first four participants again hold all the balls. Participant 1 has a black ball. Which balls are held by participants 2, 3, and 4?

Solution

- the graph of possible moves is isomorphic to a circle (since 5 and 12 are coprime),
- but the balls get passed around, so swapping the place of any two balls is not allowed. Therefore, the balls will stay in their initial (cyclic) order:

