Slide Rules

Prepared by Mark on February 25, 2025

Instructor's Handout

This file contains solutions and notes.
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> Dad says that anyone who can't use a slide rule is a cultural illiterate and should not be allowed to vote.
>
> *Have Space Suit — Will Travel, 1958*

Part 1: Logarithms

Definition 1:
The *logarithm* is the inverse of the exponent. That is, if $b^p = c$, then $\log_b c = p$.
In other words, $\log_b c$ asks the question "what power do I need to raise b to to get c?"

In both b^p and $\log_b c$, the number b is called the *base*.

Problem 2:
Evaluate the following by hand:

A: $\log_{10}(1000)$

B: $\log_2(64)$

C: $\log_2(\frac{1}{4})$

D: $\log_x(x)$ for any x

E: $log_x(1)$ for any x

Definition 3:
There are a few ways to write logarithms:

$\log x = \log_{10} x$
$\lg x = \log_{10} x$
$\ln x = \log_e x$

Definition 4:
The *domain* of a function is the set of values it can take as inputs.
The *range* of a function is the set of values it can produce.

For example, the domain and range of $f(x) = x$ is $\mathbb{R}$, all real numbers.
The domain of $f(x) = |x|$ is $\mathbb{R}$, and its range is $\mathbb{R}^+ \cup \{0\}$, all positive real numbers and 0.

Note that the domain and range of a function are not always equal.

Problem 5:
What is the domain of $f(x) = 5^x$?
What is the range of $f(x) = 5^x$?

Problem 6:
What is the domain of $f(x) = \log x$?
What is the range of $f(x) = \log x$?

Problem 7:
Prove the following identities:

A: $\log_b(b^x) = x$

B: $b^{\log_b x} = x$

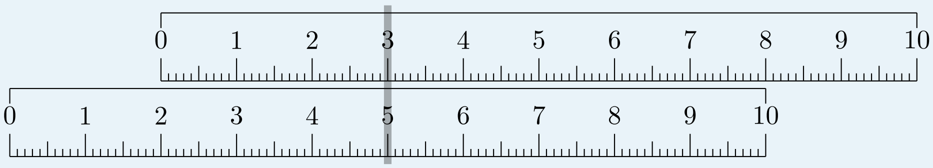
C: $\log_b(xy) = \log_b(x) + \log_b(y)$

D: $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$

E: $\log_b(x^y) = y \log_b(x)$

Note for Instructors

A good intro to the following sections is the linear slide rule:



$2 + 3 = 5$

Take two linear rulers, offset one, and you add.
If you do the same with a log scale, you multiply!

Note that the slide rules above start at 0.

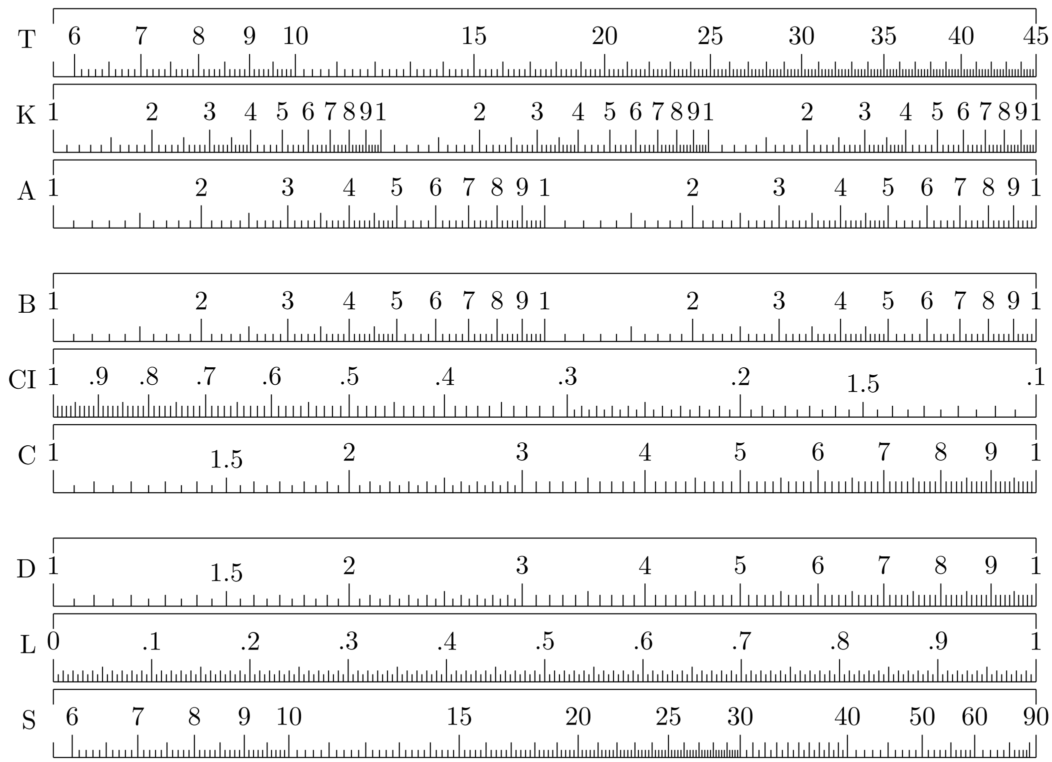
After assembling the paper slide rule, you can make a visor with some transparent tape. Wrap a strip around the slide rule, sticky side out, and stick it to itself to form a ring. Cover the sticky side with another layer of tape, and trim the edges to make them straight. Use the edge of the visor to read your slide rule!

Part 2: Introduction

Mathematicians, physicists, and engineers needed to quickly solve complex equations even before computers were invented.

The *slide rule* is an instrument that uses the logarithm to solve this problem. Before you continue, cut out and assemble your slide rule.

There are four scales on your slide rule, each labeled with a letter on the left side:



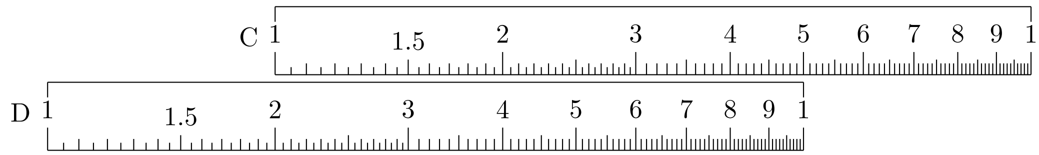
Each scale's "generating function" is on the right:

- T: tan
- K: x^3
- A,B: x^2
- CI: $\frac{1}{x}$
- C, D: x
- L: $\log_{10}(x)$
- S: sin

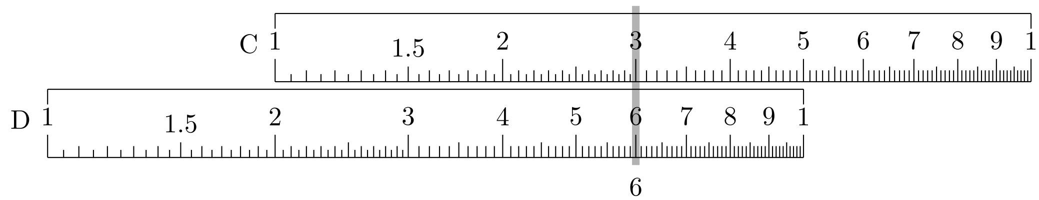
Once you understand the layout of your slide rule, move on to the next page.

Part 3: Multiplication

We'll use the C and D scales of your slide rule to multiply.
Say we want to multiply 2×3. First, move the *left-hand index* of the C scale over the smaller number, 2:



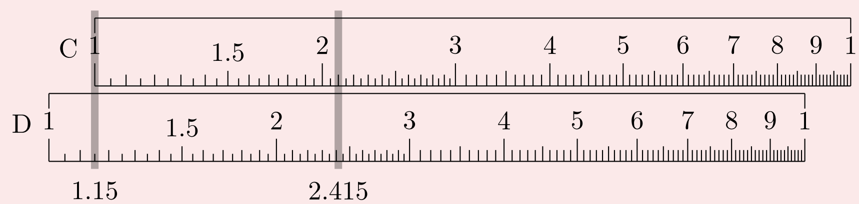
Then we'll find the second number, 3 on the C scale, and read the D scale under it:



Of course, our answer is 6.

Problem 8:
What is 1.15×2.1?
Use your slide rule.

Solution

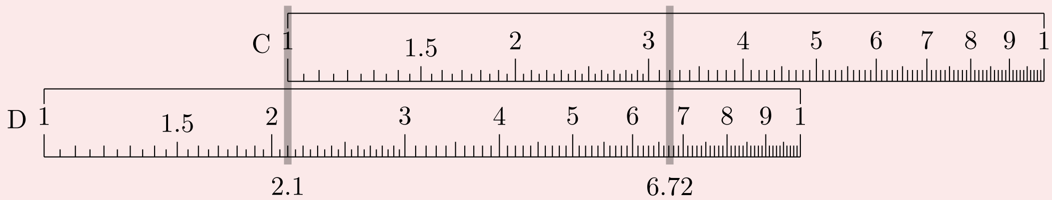


Note that your answer isn't exact. $1.15 \times 2.1 = 2.415$, but an answer accurate within two decimal places is close enough for most practical applications.

Look at your C and D scales again. They contain every number between 1 and 10, but no more than that. What should we do if we want to calculate 32×210?

Problem 9:
Using your slide rule, calculate 32×210.

Solution



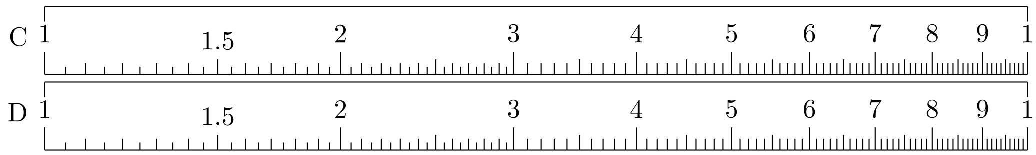
Placing the decimal point correctly is your job.
$10^1 \times 10^2 = 10^3$, so our final answer is $6.72 \times 10^3 = 672$.

Problem 10:
Compute the following:
A: 1.44×52
B: 0.38×1.24
C: $\pi \times 2.35$

Solution

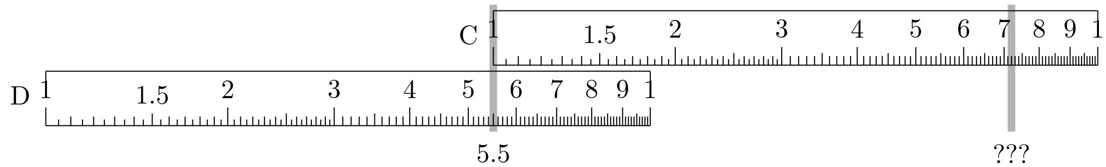
A: $1.44 \times 52 = 74.88$
B: $0.38 \times 1.24 = 0.4712$
C: $\pi \times 2.35 = 7.382$

Problem 11:
Note that the numbers on your C and D scales are logarithmically spaced.



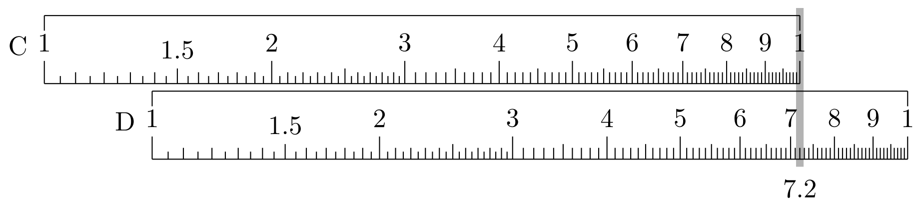
Why does our multiplication procedure work?

Now we want to compute 7.2×5.5:

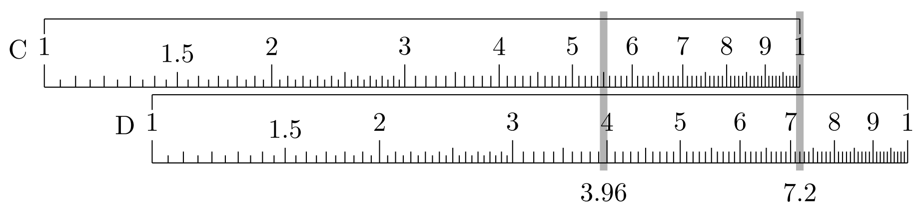


No matter what order we go in, the answer ends up off the scale. There must be another way.

Look at the far right of your C scale. There's an arrow pointing to the 10 tick, labeled *right-hand index*. Move it over the *larger* number, 7.2:



Now find the smaller number, 5.5, on the C scale, and read the D scale under it:



Our answer should be about $7 \times 5 = 35$, so let's move the decimal point: $5.5 \times 7.2 = 39.6$. We can do this by hand to verify our answer.

Problem 12:
Why does this work?

Problem 13:
Compute the following using your slide rule:

A: 9×8
B: 15×35
C: 42.1×7.65
D: 6.5^2

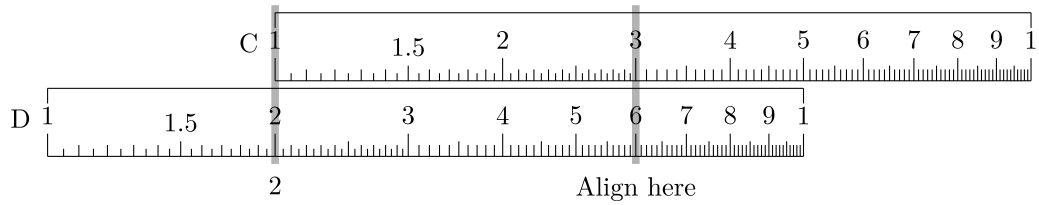
Solution

A: $9 \times 8 = 72$
B: $15 \times 35 = 525$
C: $42.1 \times 7.65 = 322.065$
D: $6.5^2 = 42.25$

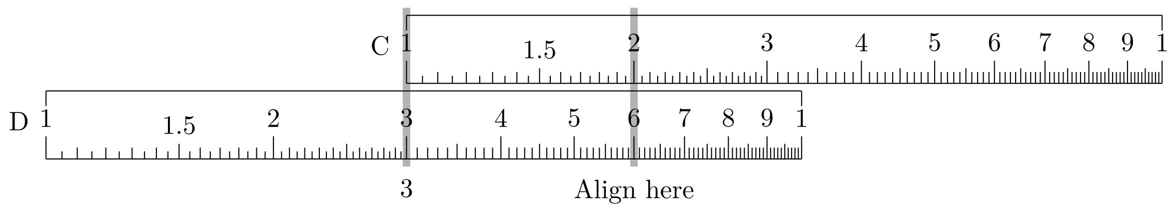
Part 4: Division

Now that you can multiply, division should be easy. All you need to do is work backwards. Let's look at our first example again: $3 \times 2 = 6$.

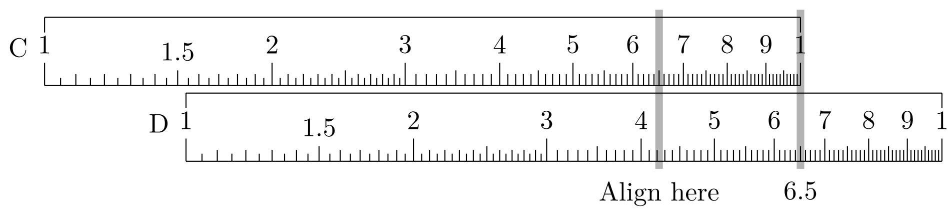
We can easily see that $6 \div 3 = 2$



and that $6 \div 2 = 3$:



If your left-hand index is off the scale, read the right-hand one.
Consider $42.25 \div 6.5 = 6.5$:



Place your decimal points carefully.

Problem 14:
Compute the following using your slide rule.

A: $135 \div 15$
B: $68.2 \div 0.575$
C: $(118 \times 0.51) \div 6.6$

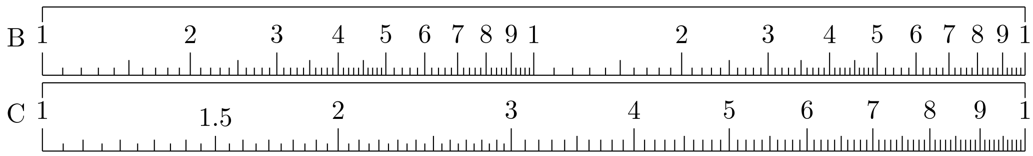
Solution

A: $135 \div 15 = 9$
B: $68.2 \div 0.575 = 118.609$
C: $(118 \times 0.51) \div 6.6 = 9.118$

Part 5: Squares, Cubes, and Roots

Now, take a look at scales A and B, and note the label on the right: x^2. If C, D are x, A and B are x^2, and K is x^3.

Finding squares of numbers up to ten is straightforward: just read the scale.
Square roots are also easy: find your number on B and read its pair on C.



Problem 15:
Compute the following.

A: 1.5^2
B: 3.1^2
C: 7^3
D: $\sqrt{14}$
E: $\sqrt[3]{150}$

Solution

A: $1.5^2 = 2.25$
B: $3.1^2 = 9.61$
C: $7^3 = 343$
D: $\sqrt{14} = 3.74$
E: $\sqrt[3]{150} = 5.313$

Problem 16:
Compute the following.

A: 42^2
B: $\sqrt{200}$
C: $\sqrt{2000}$
D: $\sqrt{0.9}$
E: $\sqrt[3]{0.12}$

Solution

A: $42^2 = 1,764$
B: $\sqrt{200} = 14.14$
C: $\sqrt{2000} = 44.72$
D: $\sqrt{0.9} = 0.948$
E: $\sqrt[3]{0.12} = 0.493$

Part 6: Inverses

Try finding $1 \div 32$ using your slide rule.
The procedure we learned before doesn't work!

This is why we have the CI scale, or the "C Inverse" scale.

Problem 17:
Figure out how the CI scale works and compute the following:

A: $\frac{1}{7}$

B: $\frac{1}{120}$

C: $\frac{1}{\pi}$

Part 7: Logarithms Base 10

When we take a logarithm, the resulting number has two parts: the *characteristic* and the *mantissa*. The characteristic is the integral (whole-numbered) part of the answer, and the mantissa is the fractional part (what comes after the decimal).

For example, $\log_{10} 18 = 1.255$, so in this case the characteristic is 1 and the mantissa is 0.255.

Problem 18:
Approximate the following logs without a slide rule. Find the exact characteristic, and approximate the mantissa.

A: $\log_{10} 20$
B: $\log_2 18$

Solution

A: $\log_{10} 20 = 1.30$
B: $\log_2 18 = 4.17$

Now, find the L scale on your slide rule. As you can see on the right, its generating function is $\log_{10} x$.

Problem 19:
Compute the following logarithms using your slide rule.
You'll have to find the characteristic yourself, but your L scale will give you the mantissa.
Don't forget your log identities!

A: $\log_{10} 20$
B: $\log_{10} 15$
C: $\log_{10} 150$
D: $\log_{10} 0.024$

Solution

Careful with number 4.

A: $\log_{10} 20 = 1.30$
B: $\log_{10} 15 = 1.176$
C: $\log_{10} 150 = 2.176$
D: $\log_{10} 0.024 = -1.6197$

Part 8: Logarithms in Any Base

Our slide rule easily computes logarithms in base 10, but we can also use it to find logarithms in *any* base.

Proposition 20:
This is usually called the *change-of-base* formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Problem 21:
Using log identities, prove Proposition 20.

Problem 22:
Approximate the following:

A: $\log_2 56$
B: $\log_{5.2} 26$
C: $\log_{12} 500$
D: $\log_{43} 134$

Solution

A: $\log_2 56 = 5.81$
B: $\log_{5.2} 26 = 1.97$
C: $\log_{12} 500 = 2.50$
D: $\log_{43} 134 = 1.30$

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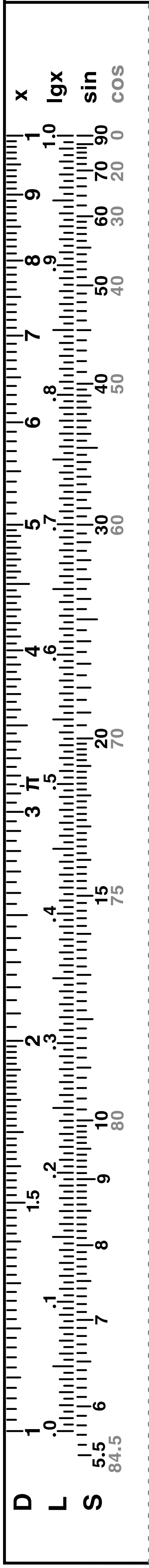
a Cut here

c Cut here



Chris Hamann and Nancy Shaw

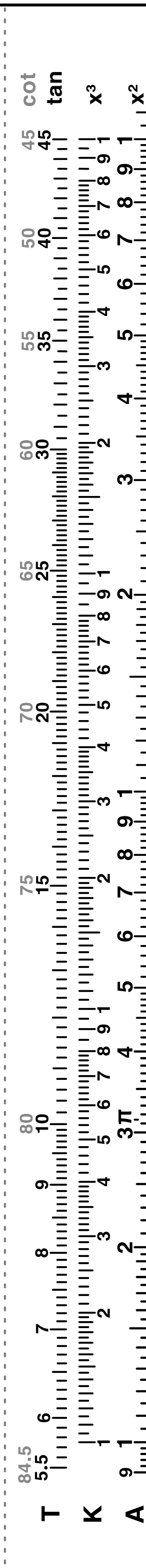
ASSEMBLY INSTRUCTIONS

1. Cut out the entire white panel (*a*). Cut along line between parts A and B (*b*), then remove excess (*c*).

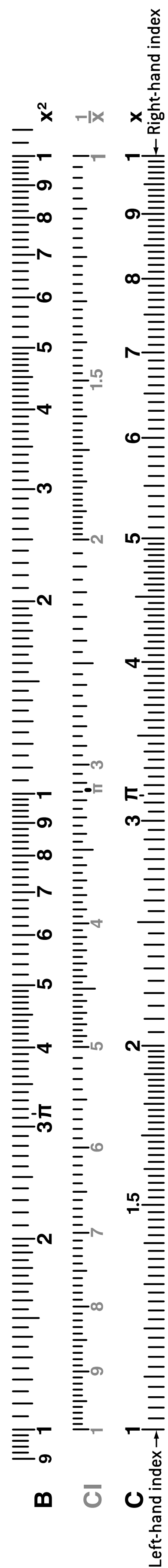
2. Fold part A along the dotted lines.

3. Slip part B into the folded part A.

Part **A**



b Cut here



(Slider)

Part **B**