

Warm-Up: Passing Balls

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Instructor's Handout

This handout contains solutions and notes.

Recompile without solutions before distributing.

Problem 1:

Twelve people are standing in a circle. Each is assigned a number between 1 and 12. Participants numbered 1, 2, 3, and 4 hold red, green, yellow, and black balls, respectively. Everyone else is empty-handed.

Each participant can pass their ball to any student that is exactly 5 positions away. Balls cannot be passed to someone who has one in hand.

After a number of passes, the first four participants again hold all the balls.

Participant 1 has a black ball. Which balls are held by participants 2, 3, and 4?

Solution:

The graph of possible moves is isomorphic to a circle (since 5 and 12 are coprime), so the order of the balls cannot be changed as they are passed around.

Therefore, the balls will stay in their initial (cyclic) order:

