

Warm-Up: Adders

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Instructor's Handout

This file contains solutions and notes.
Compile with the “nosolutions” flag before distributing.
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Problem 1:

Fill the following binary addition table.

Hint: s is “sum,” c is “carry”

a	b	s	c
0	0	?	?
0	1	?	?
1	0	?	?
1	1	?	?

Problem 2:

Draw a logic circuit that satisfies the above table.

This is called a *half adder*.

Hint: You should need exactly two gates.

Solution

$$s = a \text{ xor } b$$
$$c = a \text{ and } b$$

Definition 3:

A *full adder* is similar to a half adder, but it has an extra input:

a full adder takes a , b , and c_{in} , and produces s and c_{out} .

Hint: c_{in} is “carry in”

Problem 4:

Use two half adders to construct a full adder.

Solution

$$s_1, c_1 = \text{HA}(a, b)$$
$$s_2, c_2 = \text{HA}(s_1, c_{\text{in}})$$
$$s_{\text{out}} = s_2$$
$$c_{\text{out}} = \text{OR}(c_1, c_2)$$

Of course, the class should just draw the circuit.

Problem 5:

How can we add two four-bit binary numbers using the full adder?
We want a four-bit output sum and a one-bit c_{out} .

Problem 6:

Say that all basic logic gates need $1u$ of time to fully switch states.

Note: This is called *gate delay*

How much time does a full adder need to fully switch states?

How about your circuit from Problem 5?

Problem 7: Bonus

Design a faster solution to Problem 5.