Warm-Up: Passing Balls

Prepared by Mark on January 24, 2025

Problem 1:
Twelve people are standing in a circle. Each is assigned a number between 1 and 12.
Participants numbered 1, 2, 3, and 4 hold red, green, yellow, and black balls, respectively.
Everyone else is empty-handed.

Each participant can pass their ball to any student that is exactly 5 positions away.
Balls cannot be passed to someone who has one in hand.

After a number of passes, the first four participants again hold all the balls.
Participant 1 has a black ball. Which balls are held by participants 2, 3, and 4?

Solution

- the graph of possible moves is isomorphic to a circle (since 5 and 12 are coprime),
- but the balls get passed around, so swapping the place of any two balls is not allowed.
  Therefore, the balls will stay in their initial (cyclic) order: