Newton's Laws of Motion

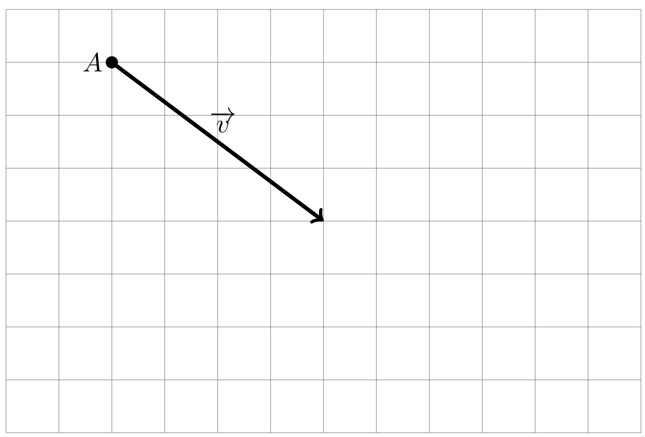
Prepared by Mark on February 15, 2026
Based on a handout by Oleg Gleizer

Part 1: Newton's First Law

If the net force acting on an object is zero, the velocity of that object does not change. Conversely, if the velocity of an object doesn't change, the net force acting on it is zero.

In the context of vectors, the "net force" is the sum of all the force vectors acting on the object. "Speed" is the length (or *magnitude*) of the velocity vector.

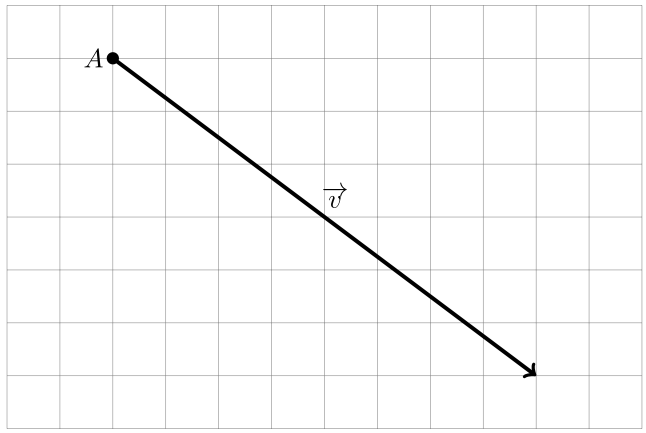
Problem 1:
There are no forces acting on the object A below. The current velocity of the object, in meters per second, is represented by the vector $\overrightarrow{v}$. Draw the position of the object two seconds later.



The sides of the grid squares on the picture above are one metre long. What is the speed of the object?

What distance would the object cover in two seconds?

Problem 2:
There are no forces acting on the object A below. The current velocity of this object, in miles per hour, is represented by the vector $\overrightarrow{v}$. Draw the position of the object half an hour later.



The sides of the grid squares on the picture above are ten miles long. What is the speed of this object?

What distance would the object cover in half an hour?

What distance would the object cover in three hours?

Definition 3: Acceleration
Acceleration is the rate at which velocity changes. Let's represent acceleration by the vector $\vec{a}$. If $\vec{a}$ does not change over time, then the speed of an object at time t is given by the following equation:

$$\vec{v_t} = \vec{v_0} + t\vec{a}. \tag{1}$$

Problem 4:
It takes a minivan seven seconds to accelerate from 0 to 60 miles per hour. Find its acceleration in meters per second squared.
Hint: 1 mile $\approx$ 1600 meters

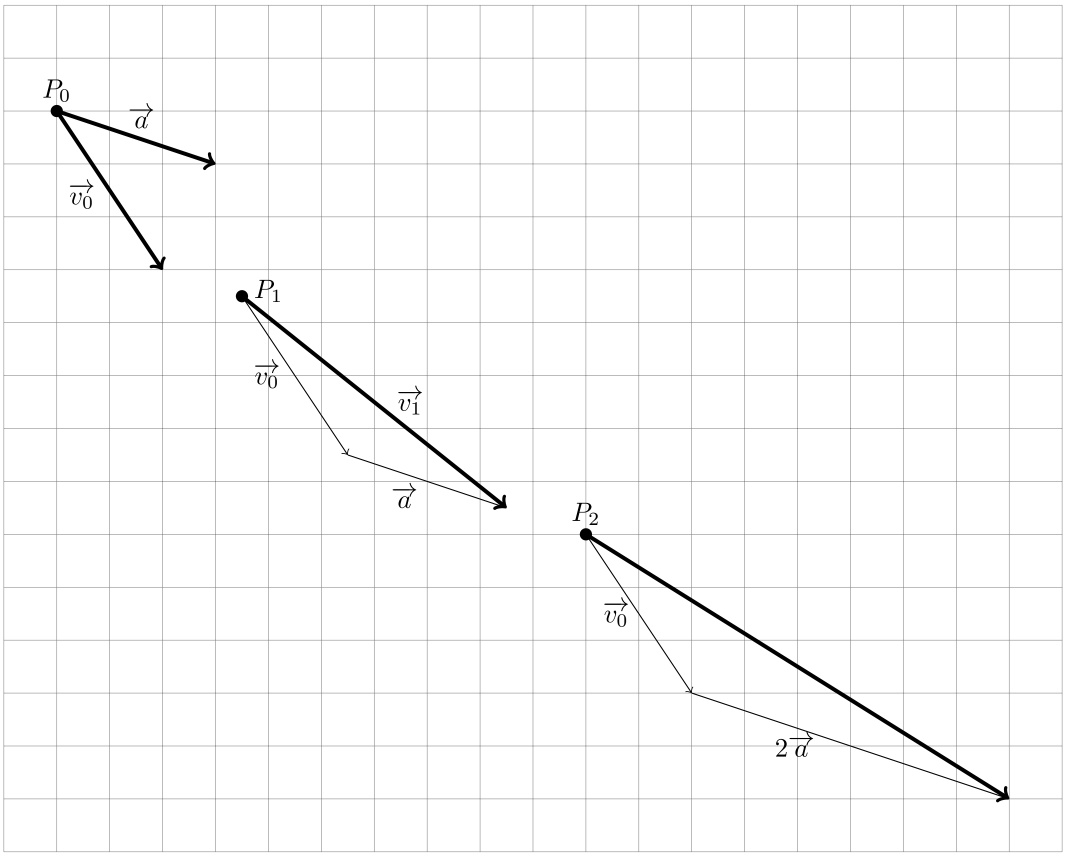
Note that in the previous problem, motion is one-dimensional (it happens on a straight line). In this case, both velocity and acceleration are one-dimensional vectors—in other words, (real) numbers! In general, velocity and acceleration are *not* numbers, but vectors. You'll see this in the next few problems.

Now, let's try a few examples with vectors:

Consider an object, currently at P_0, moving along the vector $\overrightarrow{v_0}$.
As before, let $\overrightarrow{a}$ represent the acceleration of the object. This could be caused by gravity, current, or any other constant force.

One second later, the object will be at P_1, and has the velocity vector $\overrightarrow{v_1} = \overrightarrow{v_0} + \overrightarrow{a}$.

Two seconds later, the object will be positioned at P_2 and will have the velocity vector $\overrightarrow{v_2} = \overrightarrow{v_0} + 2\overrightarrow{a}$.

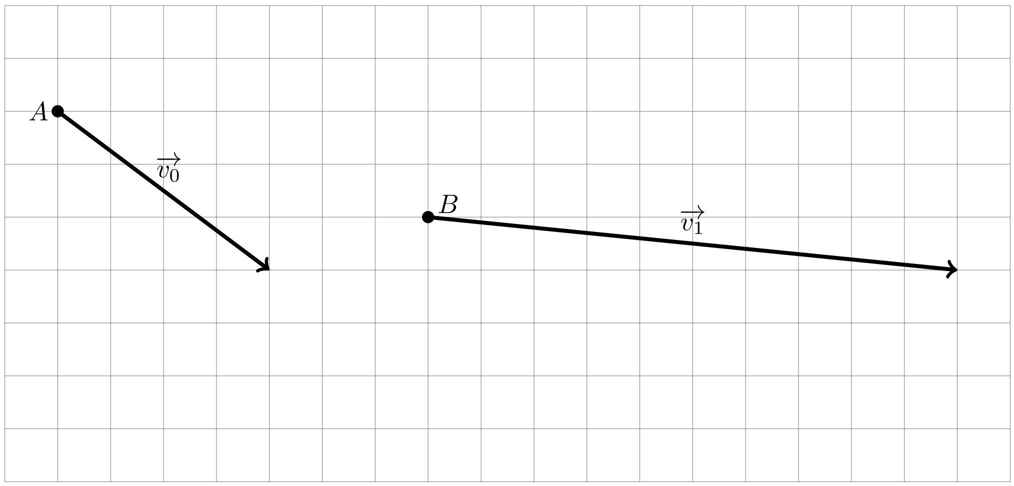


t seconds later, the object will have the velocity

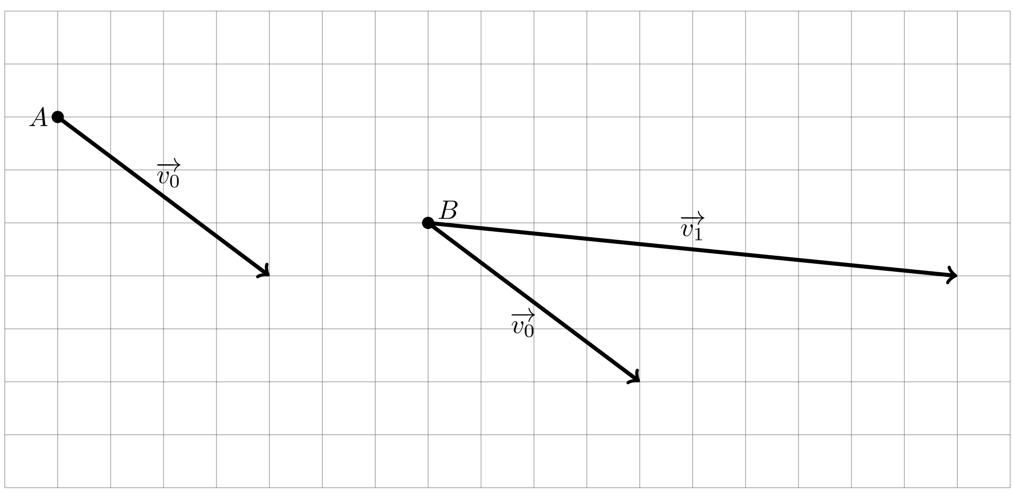
$$\overrightarrow{v_t} = \overrightarrow{v_0} + t\,\overrightarrow{a}$$

Note that the word *acceleration* has two different meanings. One is a vector, as in the example above. The other meaning is the length of the vector that represents the magnitude of the velocity change. In this case, we do not put an arrow above the letter a representing acceleration. In other words, $a = |\overrightarrow{a}|$. Similarly, speed is the length of the velocity vector, $v = |\overrightarrow{v}|$.

Example 5:
Moving with a constant acceleration, an object moves from point A to point B in one second. The velocities of the motion, in metres per second, are represented by the vectors $\overrightarrow{v_0}$ and $\overrightarrow{v_1}$. Find the acceleration vector.

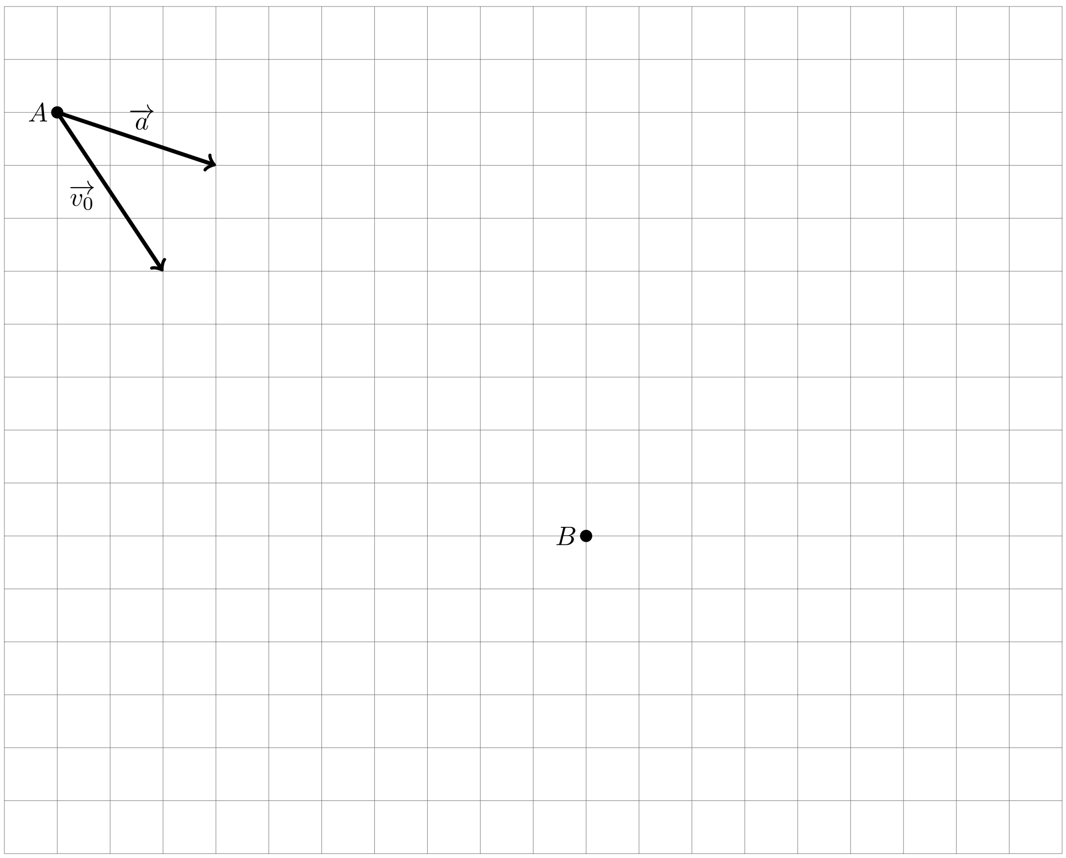


According to formula on the previous page, $\overrightarrow{v_1} = \overrightarrow{v_0} + (1 \text{ sec}) \times \overrightarrow{a}$.



Problem 6:
Complete **??**:
Draw the acceleration vector on the above diagram.

Problem 7:
At an initial moment in time, the object at point A is moving with the velocity vector $\overrightarrow{v_0}$, measured in metres per second. The constant acceleration acting on the object is represented by the vector $\overrightarrow{a}$, measured in meters per second squared. Two seconds later, the object is located at point B. Draw its velocity vector at that moment.



The side length of the grid squares on the picture above is one metre. Find the speed of the object when it is at point B.

Part 2: Newton's Second Law

The second law is simple:

$$\overrightarrow{F} = m \times \overrightarrow{a}$$

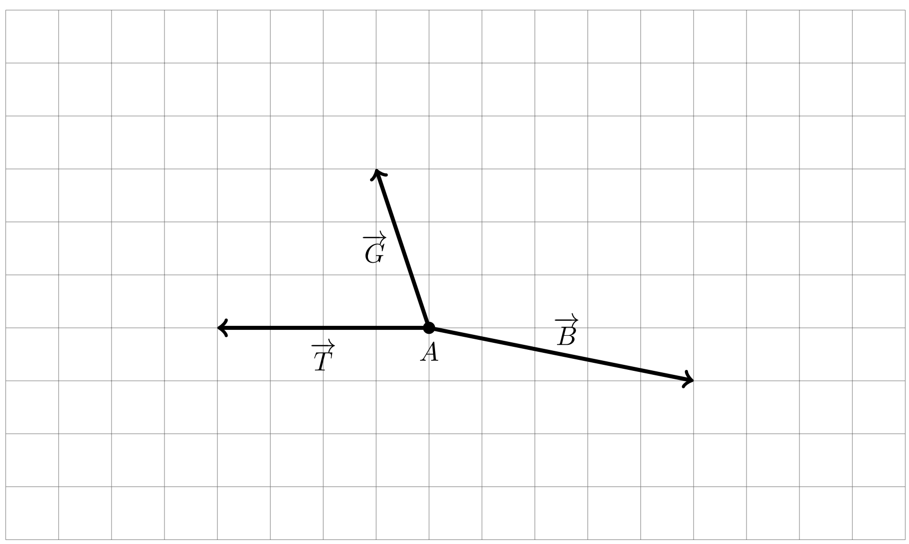
Here, $\overrightarrow{F}$ is the net force acting on an object with mass m, and $\overrightarrow{a}$ is the acceleration the object experiences as a result of this action. Mass is a measure of an object's *inertia*: the heavier an object is, the more effort it takes to change its velocity.

In civilized countries, mass is measured in grams and force is measured in *newtons*. One newton is the force it takes to accelerate 1 kg of mass to 1 meter per second. In other words,

$$1\ N = (1\ kg)(1\ \frac{m}{s})$$

Problem 8:

The *Millennium Falcon*, at point A at the moment, is trying to escape from the Death Star, which is trying to arrest the ship using its attracting beam. The thrust of the Falcon's engines, 200,000 kN in total, is represented by the vector $\overrightarrow{T}$. The force of the beam is represented by the vector $\overrightarrow{B}$. In addition, a nearby star exerts a gravitational force of $\overrightarrow{G}$ on the ship. Draw the vector of the net force acting on the vessel.



The mass of the ship is 400,000 kg. What acceleration, in metres per second squared, does the ship experience?

The purpose of this daring mission was to find out the force that the the attracting beam exerts. However, Han Solo is not particularly good with vectors. Please help him complete the mission.

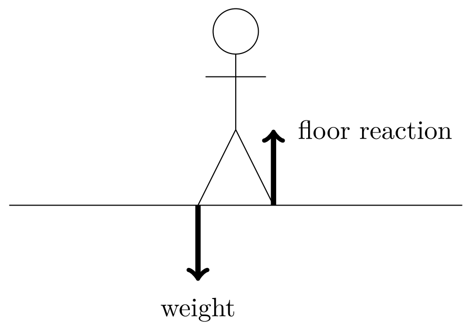
Part 3: Newton's Third Law

Newton's third law also concerns forces. It states that *every action has an equal and opposite reaction.* In other words, this means that when one object exerts force on another, the second simultaneously exerts a force equal in magnitude and opposite in direction to the force exerted on it by the first.

In fact, we saw this in our first handout!

Handout 1, Page 7

Here is an important example of an inverse vector. When you stand still, the floor pushes you up with the force opposite to the force of the gravitational pull, a.k.a. *weight*.



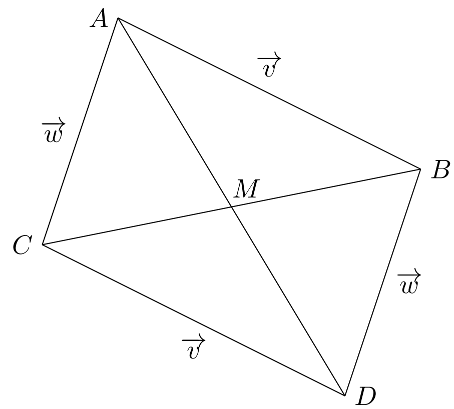
The two opposing vectors add up to the zero vector, and therefore you don't move.

Problem 9:
In a second, the truck and car on the picture below will collide in a crash test. The weight of the truck is 20 metric tons. The weight of the car is 2 metric tons. Find the ratio of the accelerations, a_t (acceleration of the truck) and a_c (that of the car), the vehicles will undergo.

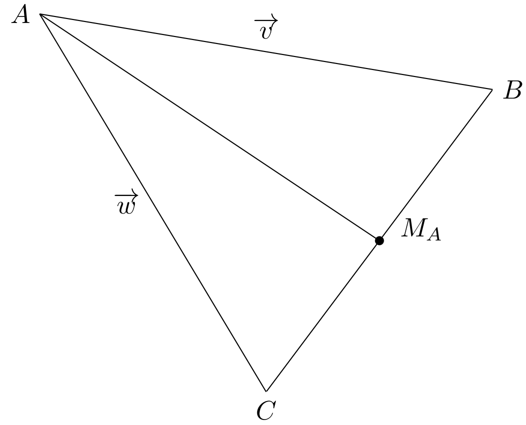
$$a_t \div a_c =$$

Part 4: Bonus Problems

Problem 10:
Use vector algebra to prove that diagonals of a parallelogram in the Euclidean plane split each other in halves.



Problem 11:
Use vector algebra to prove that all the three medians of a triangle in the Euclidean plane intersect at one point that splits each of the medians in the ratio 2:1, counting from the vertices.



Problem 12:
Find the area of an equilateral triangle with the side length a.

Problem 13:
Does there exist an equilateral triangle in the Euclidean plane such that all of its vertices have integral coordinates? Why or why not?