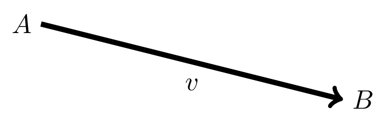
Vectors 2

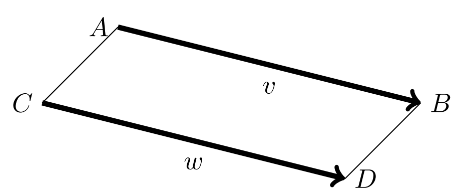
Prepared by Mark on September 25, 2025
Based on a handout by Oleg Gleizer

Part 1: Review

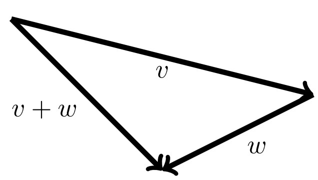
Definition 1:
A *vector* is a directed line segment. In the vector below, A is its initial point and B is its terminal point.



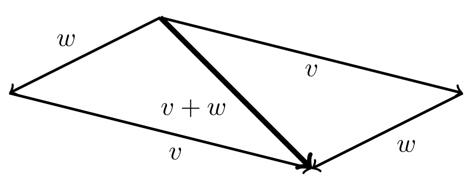
Definition 2: Equivalence
We say two vectors are equal if the quadrilateral they form is a parallelogram. In other words, two vectors are equal if they have the same length and direction.



Definition 3: Addition
To add two vectors v and w, we move w so that the initial point of w coincides with the terminal point of v. The vector originating at the initial point of v and terminating at the terminal point of w is the sum $v + w$.



Note that $v + w = w + v$. If we create a parallelogram with sides w and v (**??**), the sums create the same diagonal:

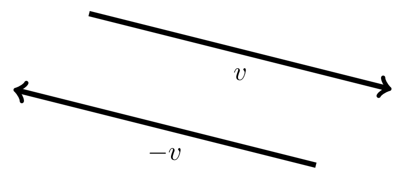


Definition 4: The Zero Vector
A vector that has coinciding initial and terminal points is called the *zero vector* and is denoted as $\overrightarrow{0}$. According to the above definition of the vector addition,

$$v + \overrightarrow{0} = \overrightarrow{0} + v = v$$

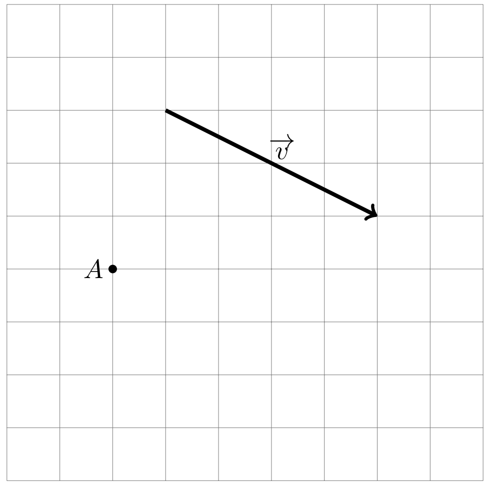
for any vector v.

Definition 5: Inverse Vectors
A vector w such that $w + v = \overrightarrow{0}$ is called the *inverse* of v and is denoted as $-v$. The vector $-v$ lies either on the same straight line as v or on a parallel one, has the same length as v, but points in the opposite direction:

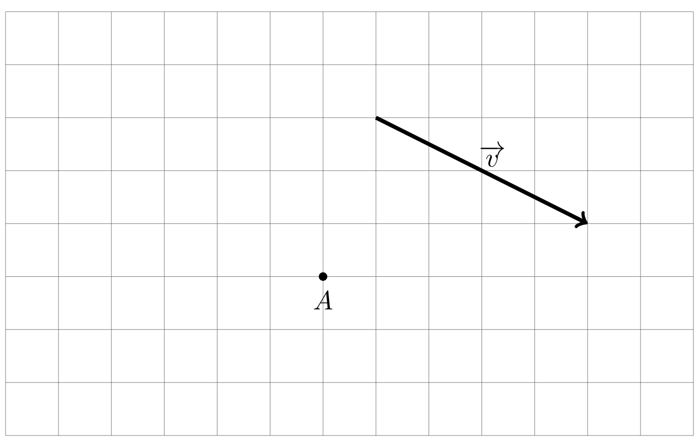


Part 2: Vectors

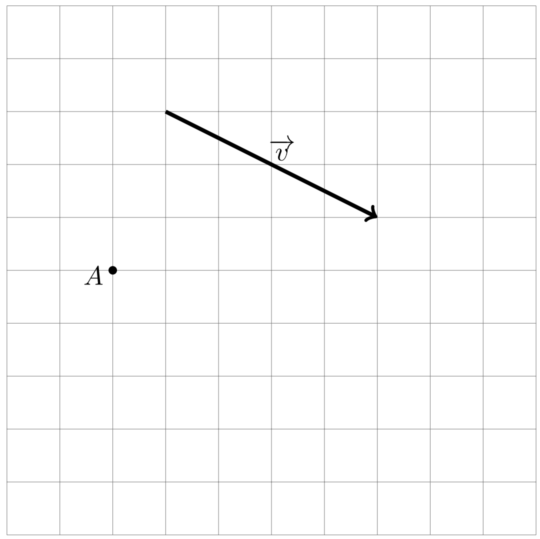
Problem 6:
For the given vector $\overrightarrow{v}$ and point A, construct the vector $\overrightarrow{w} = \overrightarrow{v}$ having A as its initial point on the graph paper below. Use the grid instead of a compass and ruler.



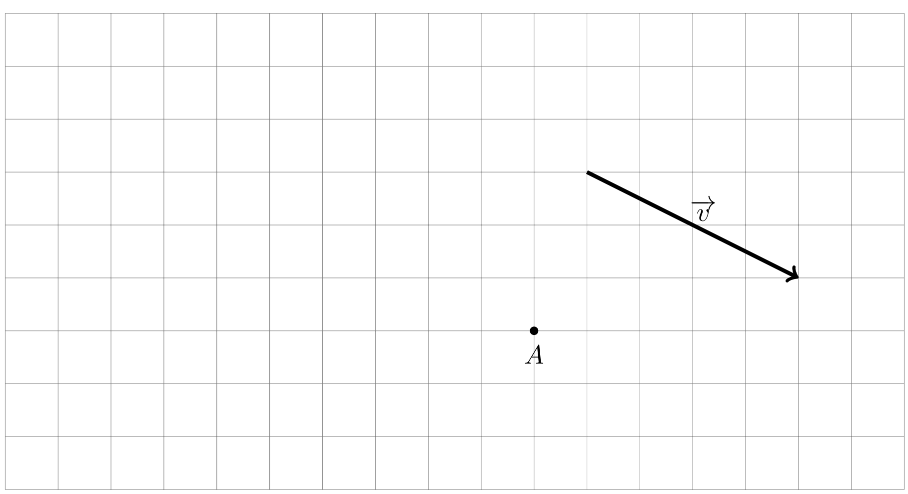
Problem 7:
For the given vector $\overrightarrow{v}$ and point A, construct the vector $\overrightarrow{w} = -\overrightarrow{v}$ having A as its initial point on the graph paper below.



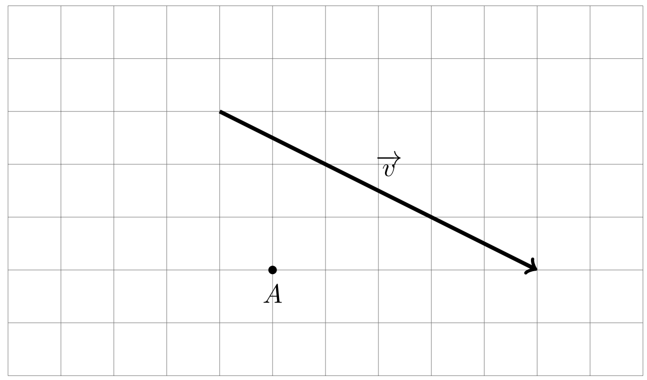
Problem 8:
For the given vector $\overrightarrow{v}$ and point A, construct the vector $\overrightarrow{w} = 1.5\overrightarrow{v}$ having A as its initial point on the graph paper below.



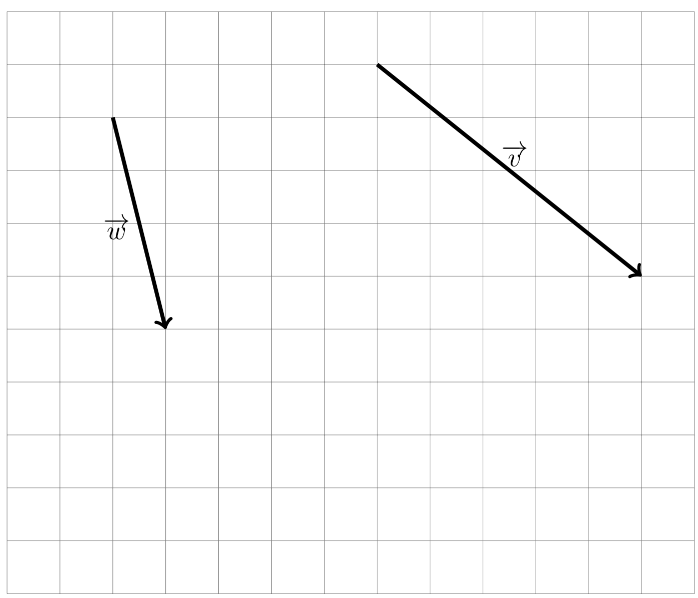
Problem 9:
For the given vector $\overrightarrow{v}$ and point A, construct the vector $\overrightarrow{w} = -2\overrightarrow{v}$ having A as its initial point on the graph paper below.



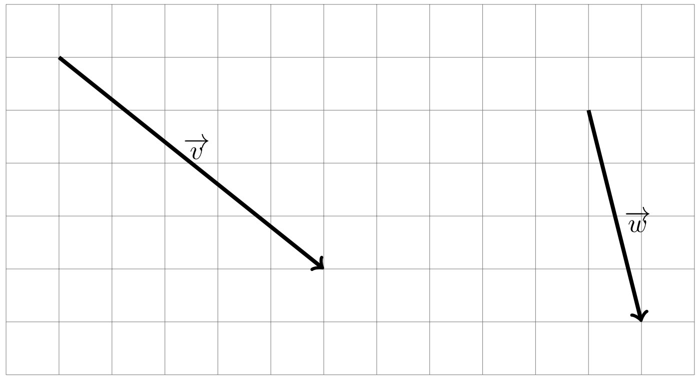
Problem 10:
For the given vector $\overrightarrow{v}$ and point A, construct the vector $\overrightarrow{w} = -\frac{1}{3}\overrightarrow{v}$ having A as its initial point on the graph paper below.



Problem 11:
For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{w} + \overrightarrow{v}$ on the graph paper below.

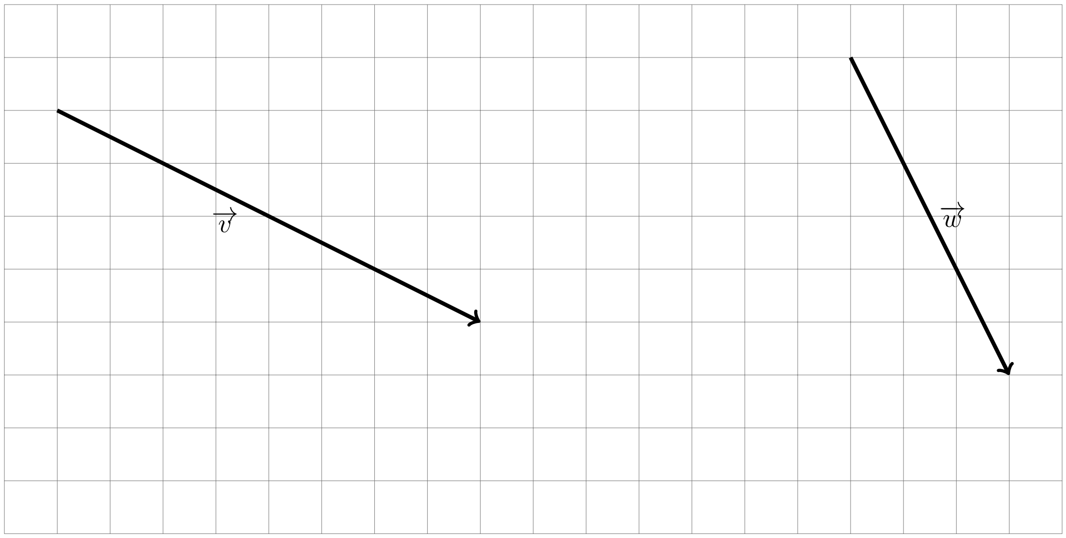


Problem 12:
For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{w} - \overrightarrow{v}$ on the graph paper below.

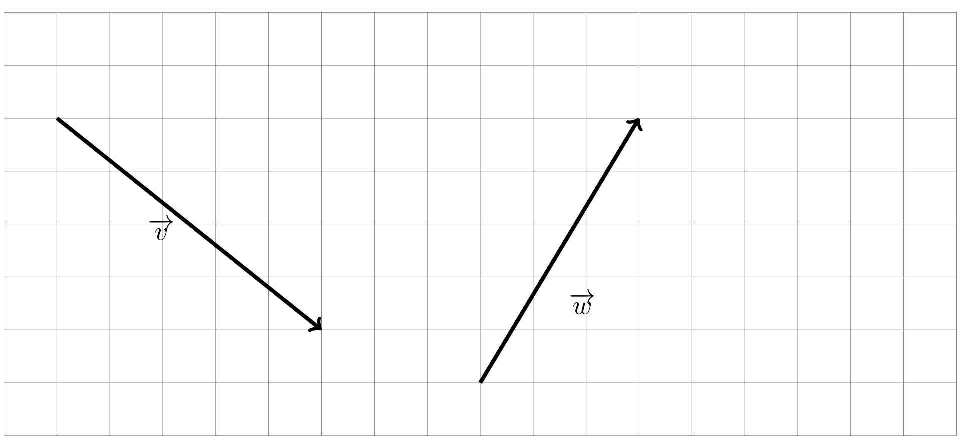


Problem 13:
For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $2\overrightarrow{v} - 3\overrightarrow{w}$ on the graph paper below.

Problem 14:
For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $1.75\overrightarrow{v} - \frac{2}{3}\overrightarrow{w}$ on the graph paper below.



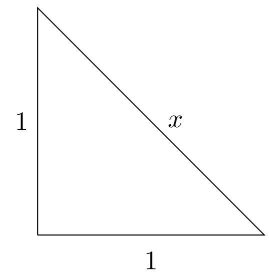
Problem 15:
For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{v} + \overrightarrow{w}$ originating at the same point as the vector $\overrightarrow{v}$ and the vector $\overrightarrow{w} + \overrightarrow{v}$ originating at the same point as the vector $\overrightarrow{w}$. Is $\overrightarrow{v} + \overrightarrow{w} = \overrightarrow{w} + \overrightarrow{v}$? Why or why not?



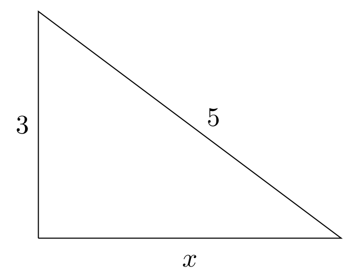
Part 3: Pythagoras' Theorem

Problem 16:
Formulate and prove the Pythagoras' theorem.

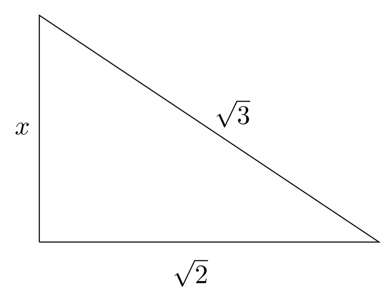
Problem 17:
Use the Pythagorean theorem to find x for the following right triangles.
a.



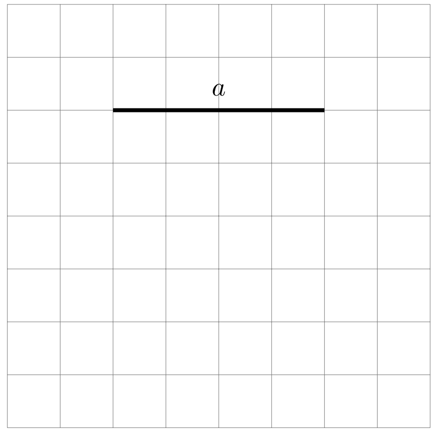
b.



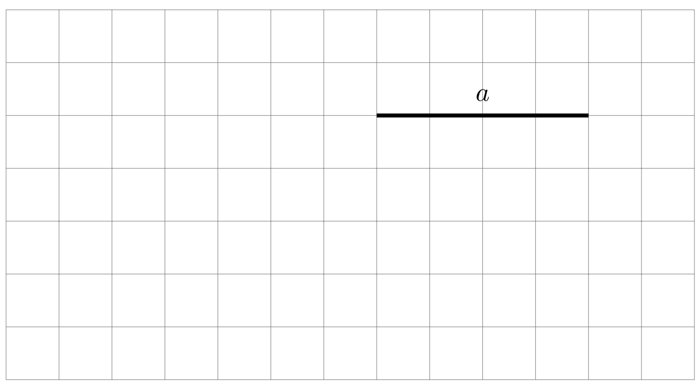
c.



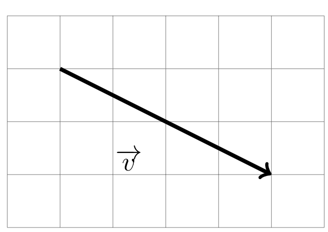
Problem 18:
Construct a segment of length $\sqrt{2}a$ on the graph paper below.



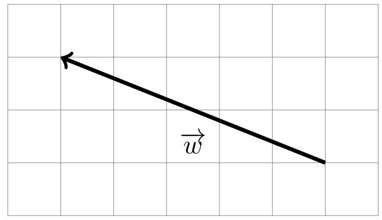
Problem 19:
Construct a segment of length $\sqrt{5}a$ on the graph paper below.



Problem 20:
The side length of a grid square below is one unit. Find the length the vector $\overrightarrow{v}$.



Problem 21:
The side length of a grid square below is three units. Find the length the vector $\overrightarrow{w}$.



Part 4: Rationals

A number is called *rational* if it can be represented as a ratio p/q of an integer p and a positive integer q such that p and q have no common factors. Otherwise, that number is called *irrational*

Problem 22:
Decide whether the following numbers are rational or irrational. In each case, give a reason.

A: $\dfrac{375}{376}$

B: 10

C: 0.5

D: -5

E: 1.2345

F: $0.111111111...$

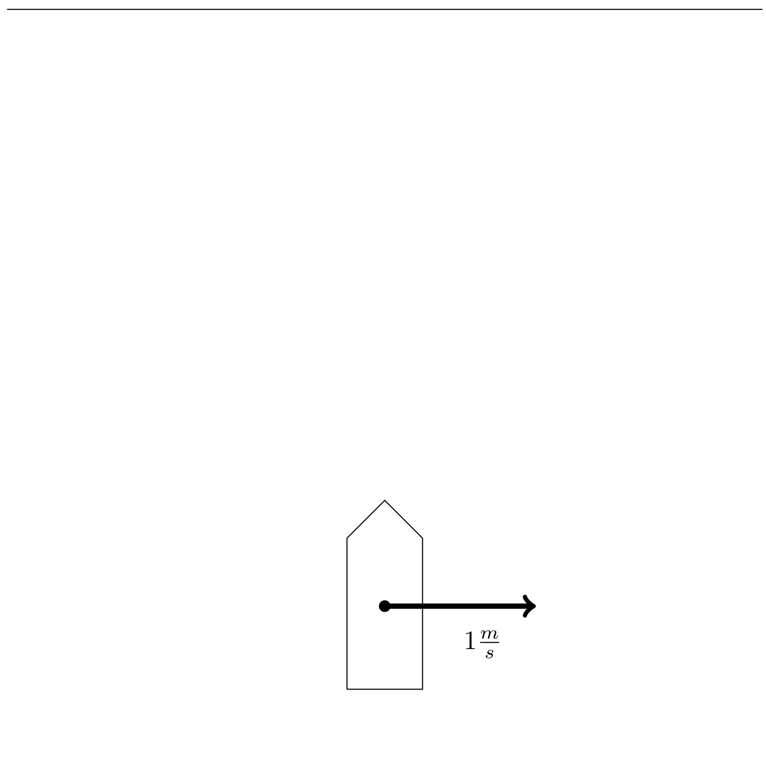
G: $\sqrt{2}$

H: $\sqrt[3]{10}$

Problem 23:
Find $\lfloor \sqrt[3]{10} \rfloor$ and $\lceil \sqrt[3]{10} \rceil$.

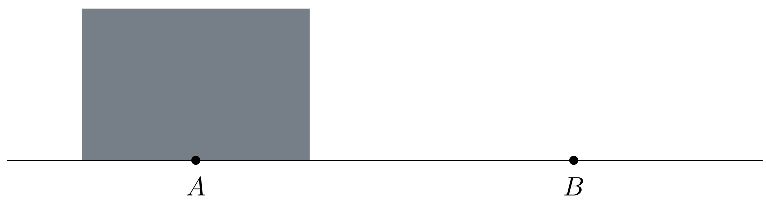
Problem 24:
Simplify $\sqrt{8}$.

Problem 25:
A man is crossing a river in a boat. The speed of the boat is three meters per second. The speed of the water in the river is one meter per second. In what direction should the man steer the boat, if he wants the vessel to move perpendicular to the banks? Construct the velocity vector.



The width of the river is $10\sqrt{2}$ meters. How long would it take the man to cross the river?

Problem 26:
You need to slide a heavy box over the floor from point A to point B. The box is about twice as tall as you are. Which way is easier, to push or to pull? Why?



Problem 27:
The dot on the picture below represents a spaceship. There are three forces acting on the ship. $\overrightarrow{T}$ is the thrust of the ship's engine. $\overrightarrow{P}$ is the gravitational pull of the neighboring planet. $\overrightarrow{S}$ is the gravitational pull of the planet's home star. You are the captain. Use a compass and a ruler to figure out where the resulting force would steer the ship.