Retrograde Analysis

Prepared by Mark on June 9, 2026
Based on books[1] by Raymond Smullyan

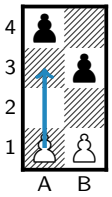
Instructor's Handout

Part 1: Introduction

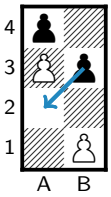
To solve the problems in this handout, you mustn't be a chess master—you just need to know how the pieces move. I'd expect that you're all familiar with the basic rules of chess (ask questions if you aren't!). The odd ones are listed below.

Board orientation:
The bottom-left square of a chessboard is *always* black.

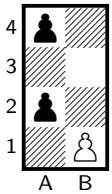
Starting pawns & en passant:
A pawn may move two squares on its first turn.
An opposing pawn may capture this pawn as it completes this move.
This is called an *en passant* capture (Which means "in passing" in French)



White moves two squares



Black captures en passant

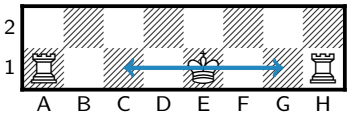


Result

Promotion:
When a pawn reaches the last row of the board, it may be promoted to any other piece.
(Except a king or a pawn, of course.)

Castling:
A king and rook can *castle* under the following conditions:

- No pieces are in the way
- The king has not yet moved
- The rook has not yet moved
- The king is not in check
- The king does not move through check while castling



Possible castle directions

Queenside castle result

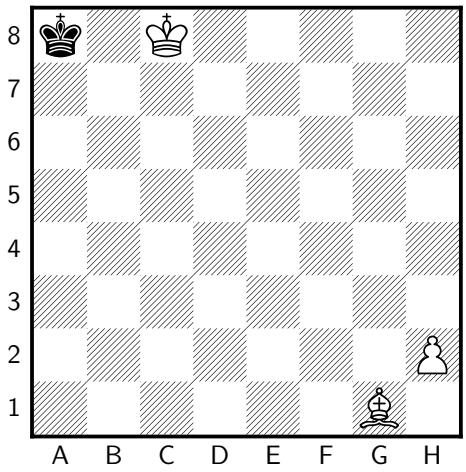
Kingside castle result

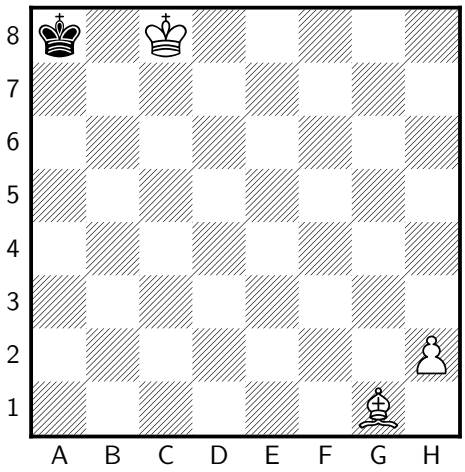
[1] Most of the easy problems in this handout are from *The Chess Mysteries of Sherlock Holmes*. The rest are from *The Chess Mysteries of the Arabian Knights*.

Part 2: Simple problems

Problem 1: A little exercise
Difficulty: ★★★★★

Black has just moved in the game below. White started on the south side of the board.
What was Black's last move, and what was White's last move?
Note: The boards below are identical copies. Scribble to your heart's content.
There a few empty boards at the end of this handout as well.





Solution

It's pretty clear that Black just moved out of check from A7.

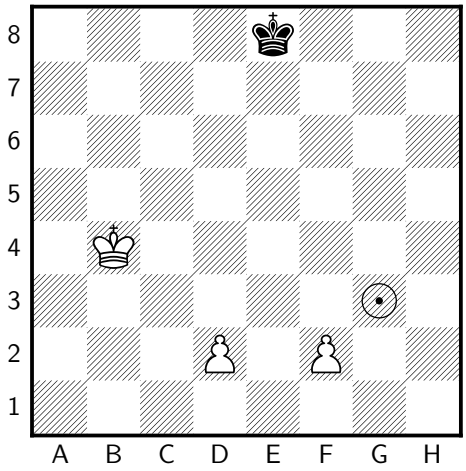
How did White deliver this check? The bishop couldn't have moved to G1, so this check must have been discovered by another piece. Since there are no extra pieces on the board, Black must've captured this piece on his last move.

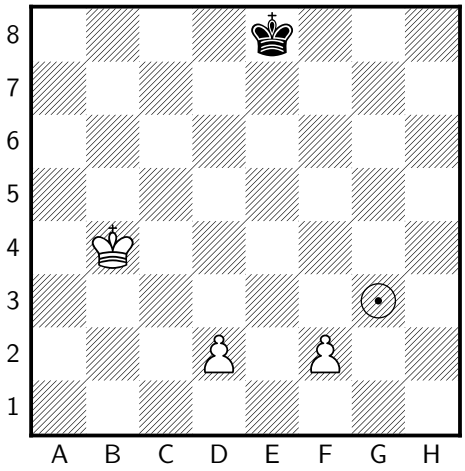
The only piece that could have moved from the white bishop's diagonal to then be captured on A8 is a knight.

Note: There are two possible answers if we don't know who started where. If Black had started on the south side of the board, that bishop could be a promoted pawn.

Problem 2: Which color?
Difficulty: ★★☆☆☆

In the game below, no pieces have moved from a black square to a white square, or from a white square to a black square. There is a pawn at G3. What color is it?
As before, White started on the bottom of the board.





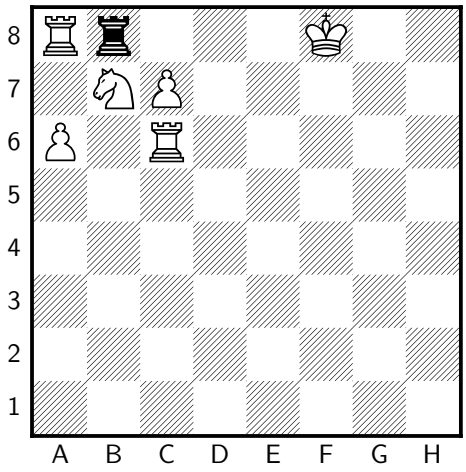
Solution

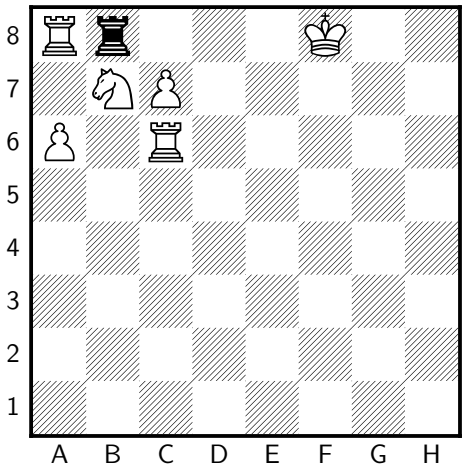
The white king is the key to this solution. How did it get off of E1?
It must have castled kingside—castling queenside would move a rook from black to white.

Now, the white king is on G1. How did it get out of there?
It's must have moved through H2 and G3, which would be impossible if the mystery pawn on G3 was white. Therefore, that pawn must be black.

Problem 3: Invisible, but not invincible
Difficulty: ★★☆☆☆

The black king has turned himself invisible. Unfortunately, his position is hopeless.
Mate the king in one move.





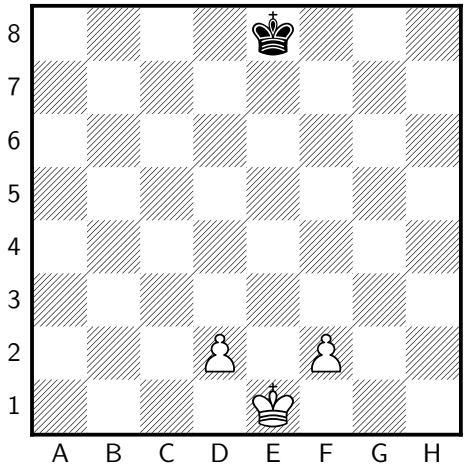
Solution

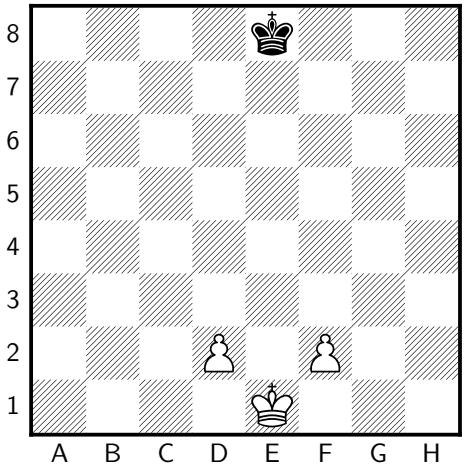
Since it is White's move, Black cannot be in check.
So, either White is in check or the black king is on C8.
If White is in check, Black must have administered this check by moving from C8 to D7.
Therefore, the black king must be on C8 or D7.

If we capture the black rook on B8 with the pawn on C7 and promote it to a knight, the black king will be in checkmate regardless of his position.

Problem 4: An empty board
Difficulty: ★★☆☆☆

In the game below, no pieces have moved from a black square to a white square, or from a white square to a black square. There is one more piece on the board, which isn't shown. What color square does it stand on?





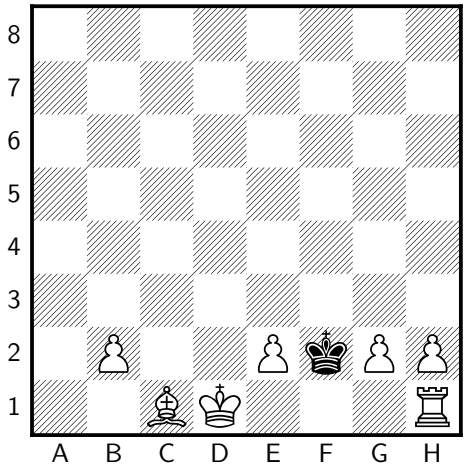
Solution

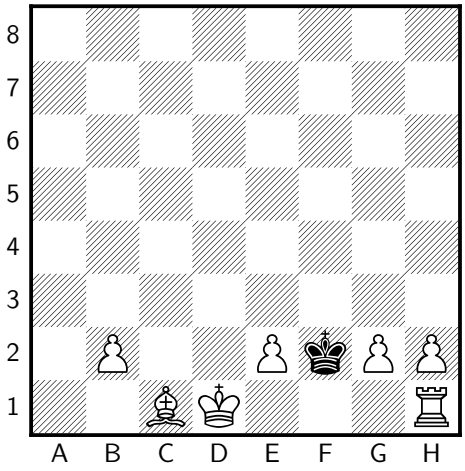
Which piece performed the last capture on a black square? It couldn't have been a white pawn, which haven't moved. It couldn't have been the white king, which is trapped; or the black king, which is restricted to white squares.

It must have been the piece we can't see, which therefore stands on a black square.

Problem 5: Promotion?
Difficulty: ★★☆☆☆

It is White's move. Have there been any promotions this game?





Solution

Since it is White's move, Black has just moved his king. Where did he move it from? Not E1, E3, F3, or G3, since that implies Black had moved into check before.

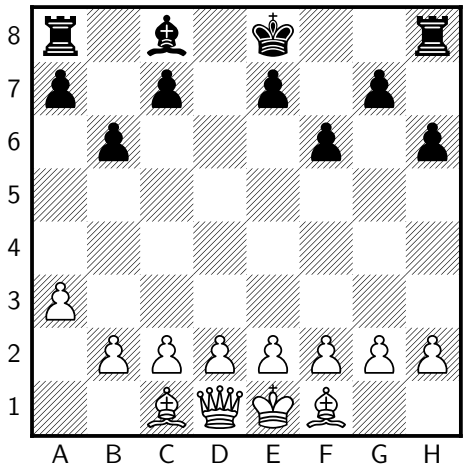
The only remaining possibilities are F1 and G1.
G1 is again impossible: how would the king get there without moving into check?
F1, therefore, is the only choice. If we place the king on F1, we see that another piece must prevent check from the white rook. This must have been a white black-square bishop, which moved to F2 to reveal that check, and was then captured by the black king.

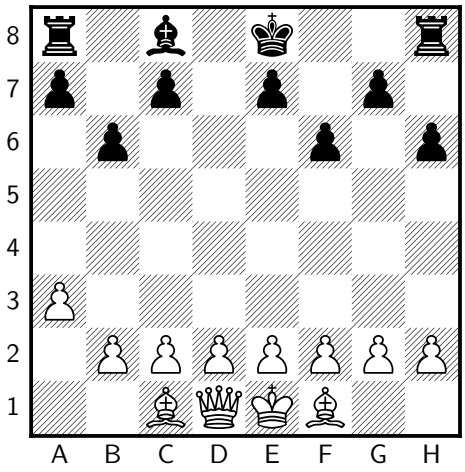
However, there is already a white black-square bishop on the board! We can get a second only by promoting a pawn, so the answer is "yes."

Problem 6: Whodunit
Difficulty: ★★☆☆☆

It is Black's move. Can Black castle?

Hint: Remember the rules of chess: you may not castle if you've moved your rook.





Solution

White's last move was with the pawn.
Black's last move must have been to capture the white piece which moved before that.

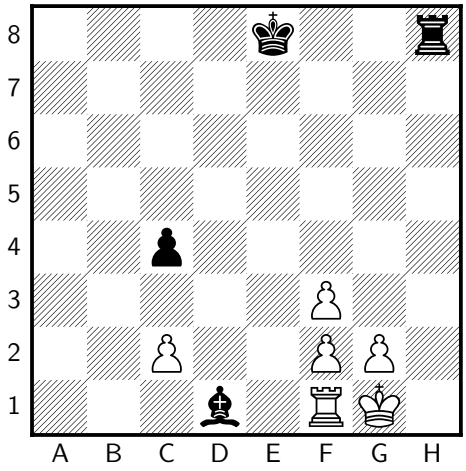
This piece would have to have been a knight, since the white rooks could not have got out onto the board. It is clear that none of the black pawns captured this knight. The black rook on A8 couldn't have captured it either, because there is no square that the knight could have moved from to get to that position.

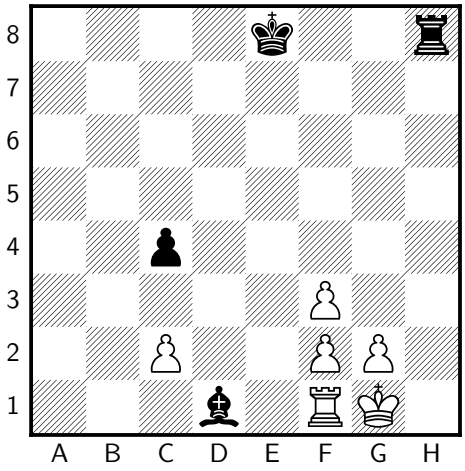
The black bishop couldn't have captured the knight either, since the only square the knight could have come from is D6, where it would have been checking the king.

So, the black king or the rook on H8 made this capture. Therefore, Black can't castle.

Problem 7: Castle contradiction
Difficulty: ★★☆☆☆

Neither Black nor White captured a piece on their last move.
It is Black's move. Show that he cannot castle?
Hint: What was White's last move? Check the cases.





Solution

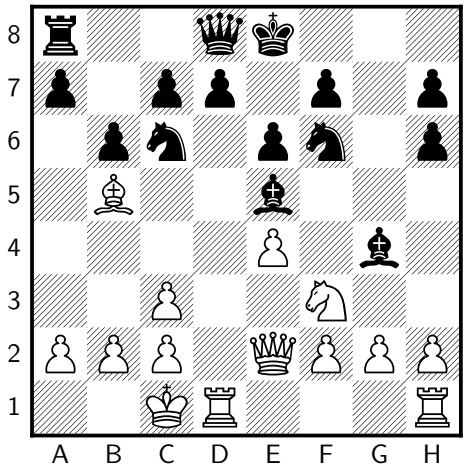
If White's last move was with the king, then the black rook moved to check him and Black can't castle.

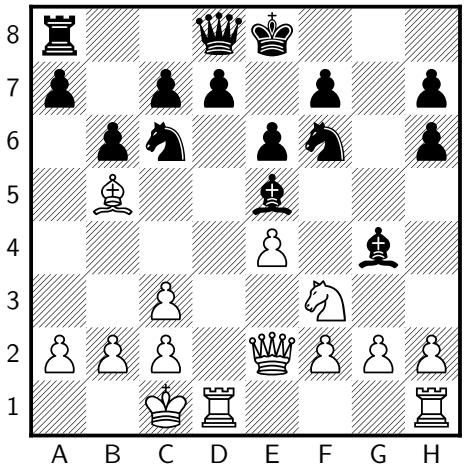
If White's last move wasn't with the king, White must have castled.
What was Black's last move?
If it was with the king or rook, Black can't castle.

It could not have been with the bishop, since then White would have had no move immediately before that. Now, suppose Black moved his pawn. Then White's preceding move must have been with the pawn from E2, capturing a piece on F3. This means that the bishop on D1 is a promoted bishop. The promoting pawn must have come from D7, passed D2, checked the white king, making it move! This contradicts our assumption that White has just castled.

Problem 8: A matter of order
Difficulty: ★★★☆☆

A black bishop captured a White piece earlier in this game.
Which bishop was it, and what did it capture?
Hint: Black and White start with 16 pieces each.





Solution

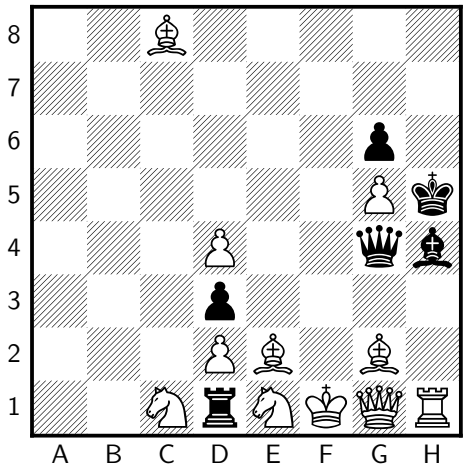
First, notice that the pawn on C3 came from D2 by capturing a piece.
This must have been a black rook, which is the only missing black piece.

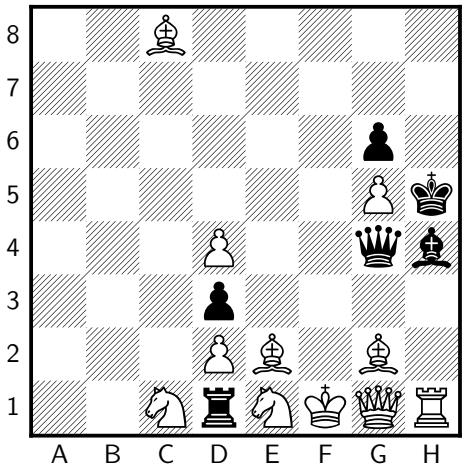
This black rook couldn't have moved there before the black pawn on G7 captured a white piece on H6. This piece couldn't have been the missing white bishop, because that bishop would still be trapped by the pawn on D2. Therefore, the missing white knight was captured on H6.

The only other missing white piece is the black-square bishop, which must have been captured by the black bishop on E5.

Problem 9: Moriarty's first
Difficulty: ★★★☆☆

No captures have been made in the last four moves.
It is White's move. What was the previous move?





Solution

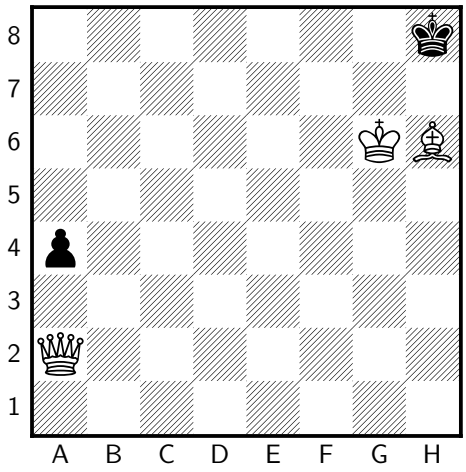
To see what the position was four moves ago, move the Black queen to E4, the knight on E1 to F3, the Black bishop to E1, and the White bishop on C8 to H3.
The following sequence of moves brought the game to the present position:

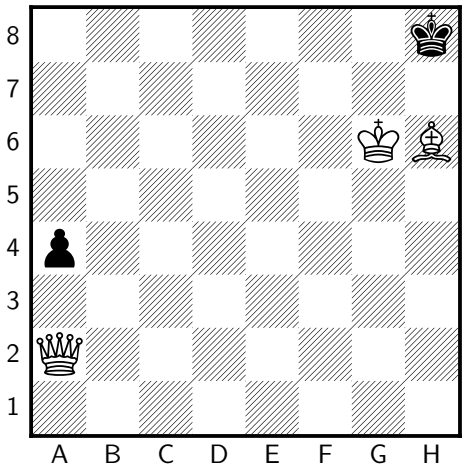
- bishop to c8, check
- bishop to h4, check
- knight to e1, check
- queen to g4.

This is the only way the present position could have arisen, so Black's last move was with the queen from E4 to G4.
Try any other last move, and you will find it impossible to play back three more moves.

Problem 10: Moriarty's second
Difficulty: ★★★☆☆

Neither the White king nor queen has moved during the last five moves, nor has any piece been captured during that time. What was the last move?





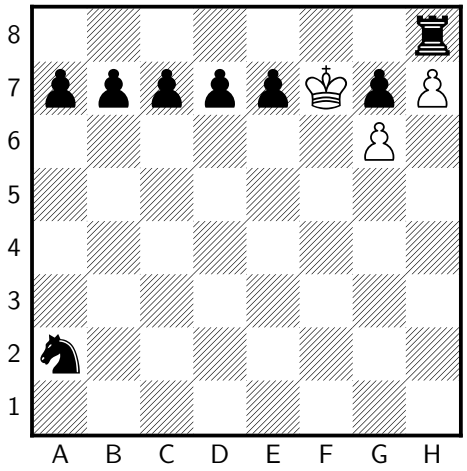
Solution

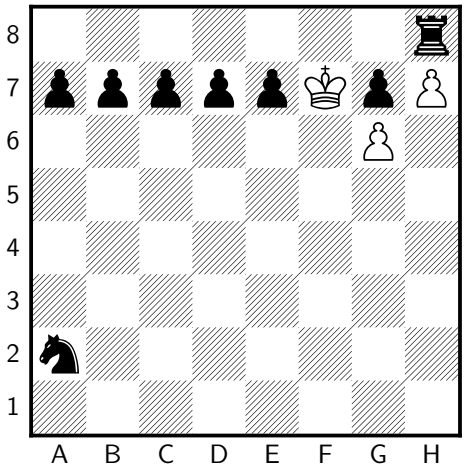
Put the Black pawn on A7, the Black king on G8, remove the White bishop, and put a White pawn on d5; this was the position five moves ago. The following sequence of moves brought the game to its present position:

- White: P-d6
- Black: K-h8
- White: P-d7
- Black: P-a6
- White: P-d8 = B
- Black: P-a5
- White: B-g5
- Black: P-a4
- White: B-h6

Problem 11: Moriarty's third
Difficulty: ★★★☆☆

No pawn has moved, nor has any piece been captured in the last five moves.
The Black king has been accidentally knocked off the board.
On what square should he stand?





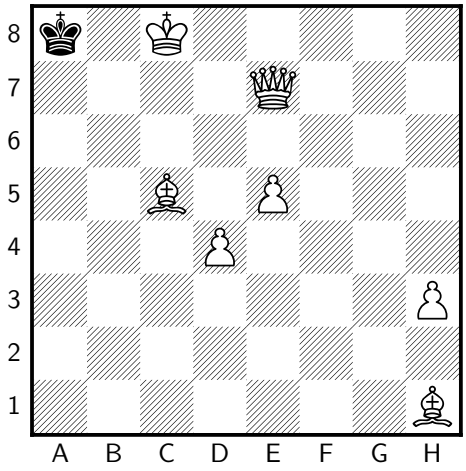
Solution

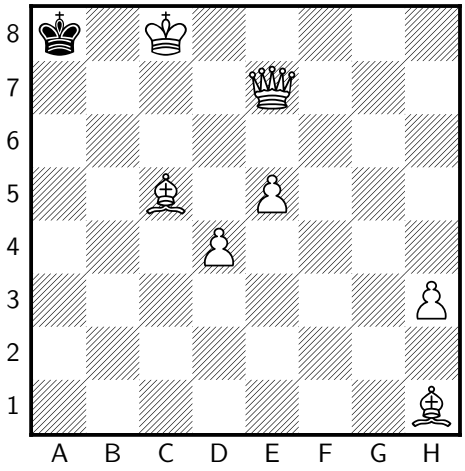
The only way to avoid a retrograde stalemate for White is by placing the Black king on C8. Black's last move was with the rook from D8, White's move before that was with his king from G8, and Black's move before that was to castle.

Part 3: Slightly harder problems

Problem 12: A matter of direction
Difficulty: ★★★☆☆

The results of a game of chess are shown below.
Did White start on the north or south side of the board?





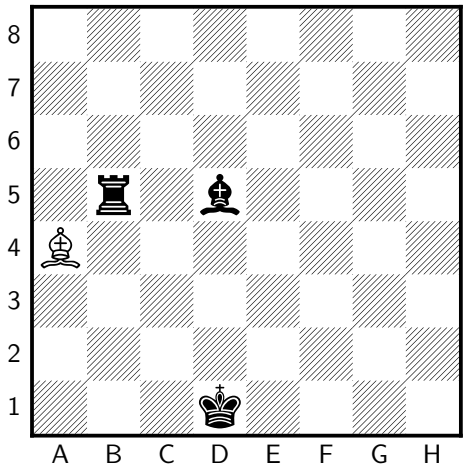
Solution

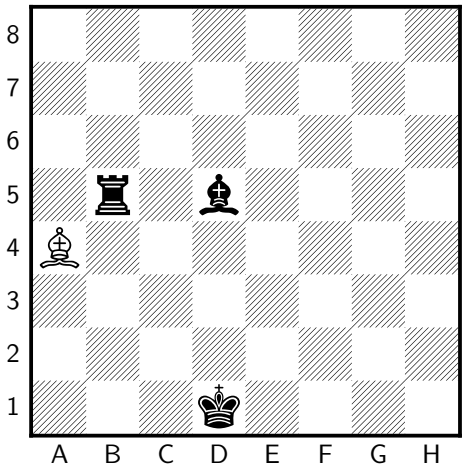
Let us first find White's last move. It wasn't with the pawns on D4 and E5, since Black wouldn't have a move before that. (Note the double-check on A7).

How, then, did White put Black in check? There are no pieces that could've uncovered this check, and the bishop on H1 couldn't have moved from anywhere. We thus see that that bishop must be a promoted pawn, proving that White started on the north side of the board.

Problem 13: Where is the king?
Difficulty: ★★★☆☆

The white king has again become invisible. Find him.
Hint: White started on the bottom. En passant.



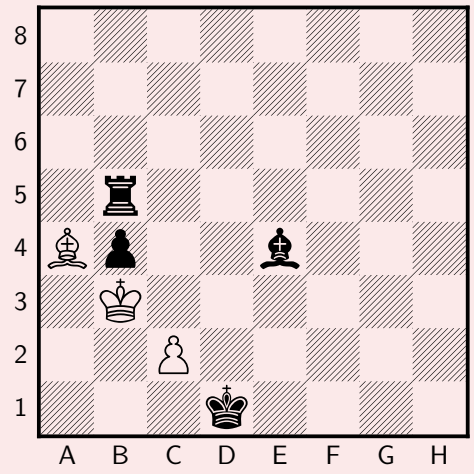


Solution

Looking at the board, we see that the white king is on B3 or Black is in check.

First, we show that the latter implies the former: assume the black king is not on B3. How did White deliver this check? Not by moving the bishop, so this check must have been discovered by the white king moving from B3. Therefore, if the white king isn't on B3 now, he was there on the previous move.

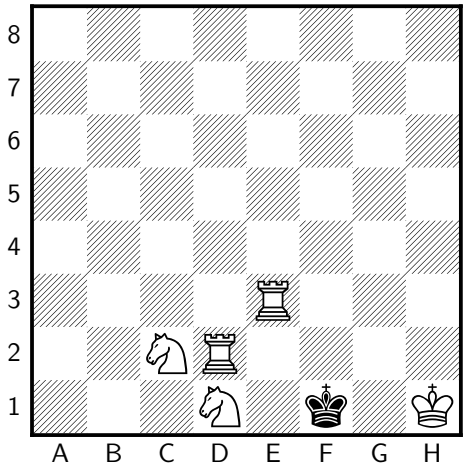
How did the white king end up on B3? That seems to be an impossible double-check from both the rook and bishop! Looking at the hint, we place a black pawn on B4 to block check from the rook, and a white pawn on C2 that this black pawn will capture. The sequence of moves is now as follows:

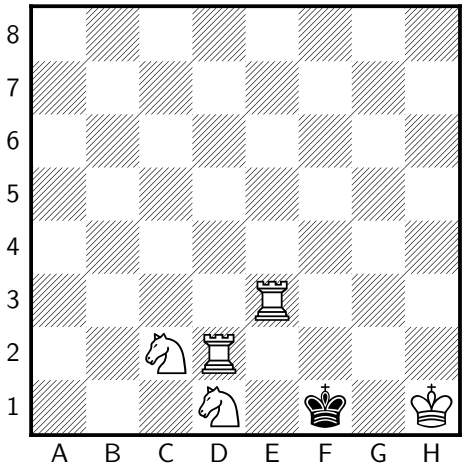


Black: E4 → D5
White: C2 → C4
Black: B4 → C3 (en passant capture)
White: B3 → C3
So, the white king must be on C3.

Problem 14: Double-checks
Difficulty: ★★★☆☆

White to move. Which side of the board did each color start on?
Hint: What was Black's last move?

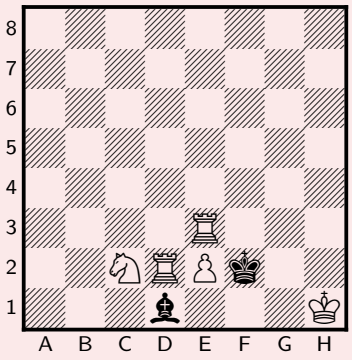




Solution

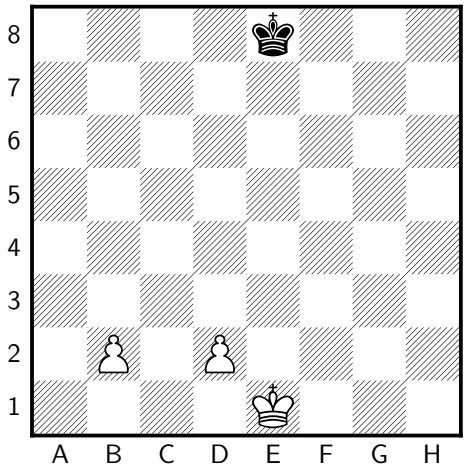
Black's last move was from F2, where his king was in double-check from both a rook and a knight. How did this happen?

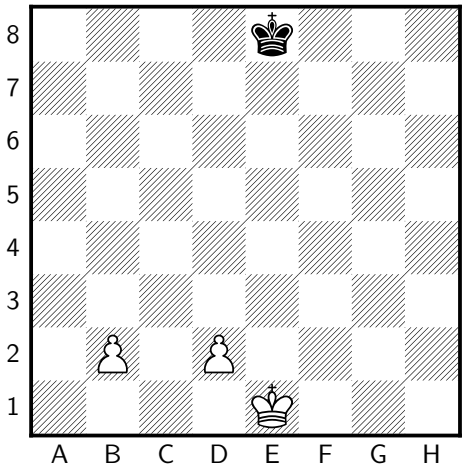
White started on the north side of the board, and put Black in check by capturing a piece on D1 with a pawn and then promoting that pawn to a knight.



Problem 15: Monochromatic
Difficulty: ★★★★☆

In the game below, no pieces have moved from a black square to a white square or from a white square to a black square. The white king has made fewer than fourteen moves.
Use this information to show that a pawn was promoted.





Solution

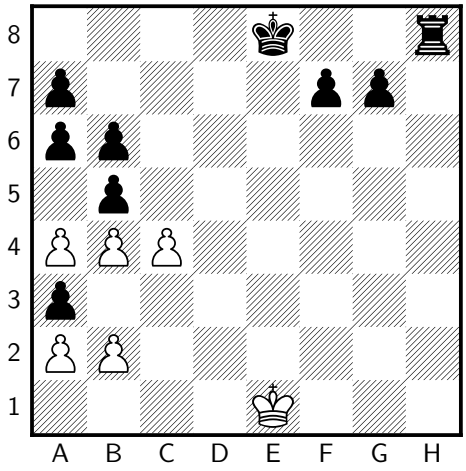
Knights always move to a different colored square, so all four missing knights must have been captured on their home square. What pieces captured them?

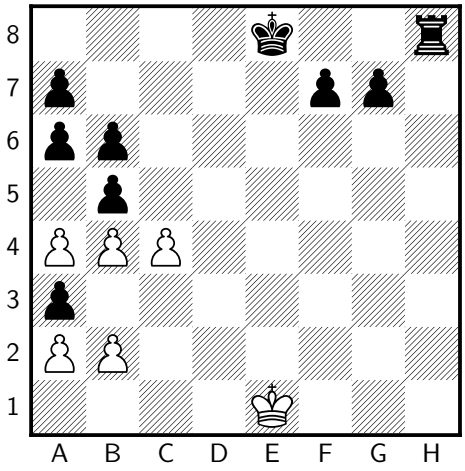
We can easily account for the white knights and the black knight on G8, but who could've captured the knight from B8? The only white pieces that can move to black squares are pawns, the Bishop (which is trapped on C1), the rook (which is stuck on column A and row 1), or the king (which would need at least 14 moves to do so).

If this knight was captured by a pawn, that pawn would be immediately promoted. If it was captured by a piece that wasn't a pawn, that piece must be a promoted pawn.

Problem 16: Superposition
Difficulty: ★★★★☆

A white pawn is missing; it is either on F2 or G2.
Where is it?

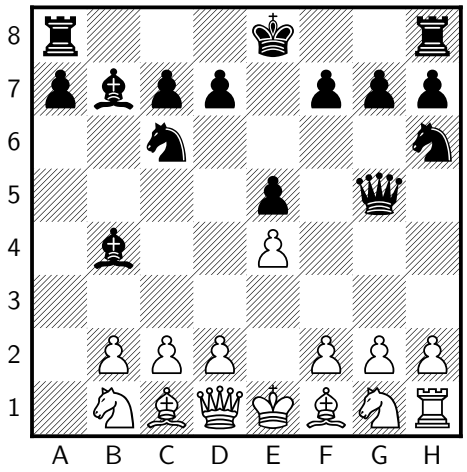


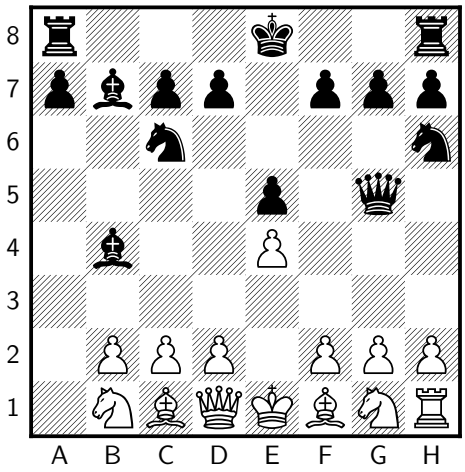


Problem 17: Possibility
Difficulty: ★★★★☆

Show that black can castle to either side.
We know the following:

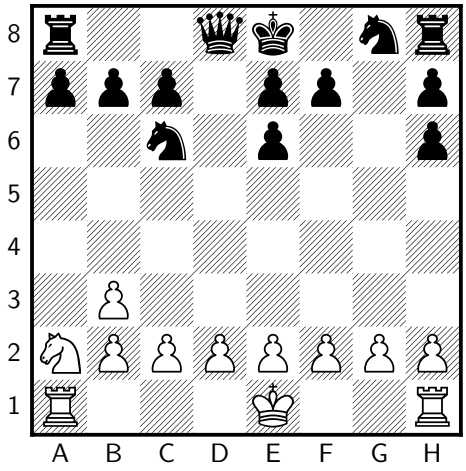
- White started the game missing one rook.
- White has not moved either knight
- No promotions have been made
- White's last move was from E2 to E4.

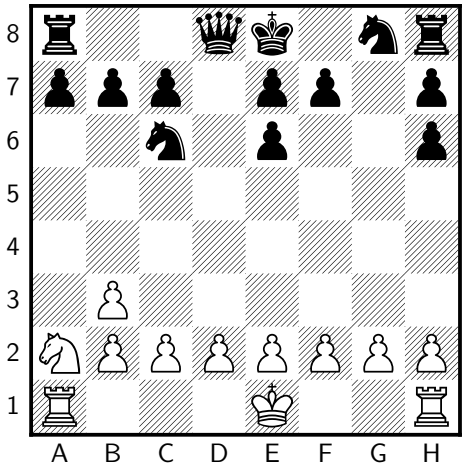




Problem 18: Kidnapping
Difficulty: ★★★★☆

On which square was the White queen captured?.





Solution

White is missing a queen, both bishops, and one knight.
The black pawns on E6 and H6 account for two captures.

Neither white bishop could've been captured by these pawns, since both are trapped by their pawns. Thus, these black pawns must have captured a queen and a knight.

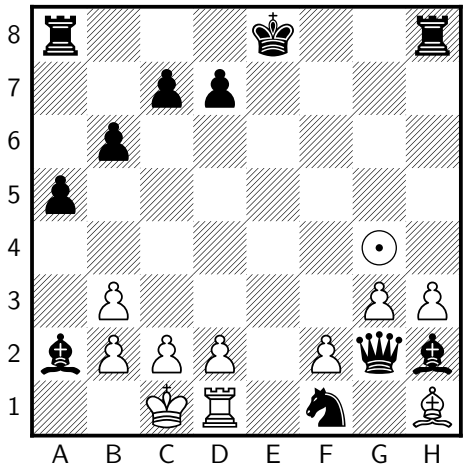
The white pawn on B3 must have captured a black bishop.
The white queen got onto the board through A2.
Therefore, the pawn on B3 made its capture before the queen escaped, and the black bishop was captured before the white queen.

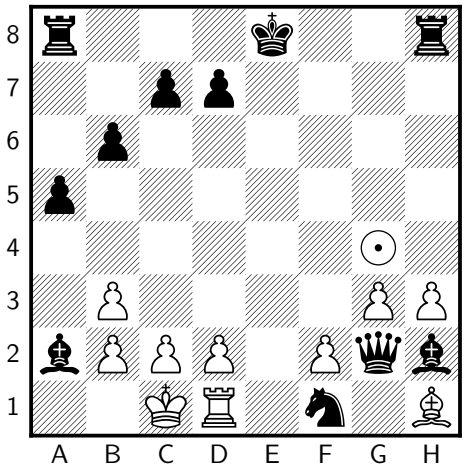
Similarly, the bishop from C8 must have been captured on B3 after the capture on E6, since it got on the board through D7.

The capture on E6 was made before the capture on B3 (black bishop), which was made before the white queen was captured. Therefore, the white queen was not captured on E6, and must have been lost on H6.

Problem 19: A missing piece
Difficulty: ★★★★★★★★

There is a piece at G4, marked with a ⊙.
What is it, and what is its color?





Solution

Part 1:
The black bishop on A2 cannot be original, since the white pawn on B3 would have prevented it from getting there. That bishop is a promoted bishop.
The black pawn it was promoted from must have come from E7, captured four pieces to get to A3, then moved to A2, and then made a capture on B1, where it was promoted.
Thus, the pawn from E7 has made five captures.

The white bishop from from C1 never left its home square (since neither of the pawns on B2 or D2 have moved), and hence was captured on C1. This makes six captures of white pieces, which tells us that the mystery piece is black.

Part 2:
White's last move could not have been with the rook from E1, which would have checked Black, nor with the king (which could only come from B1, an impossible check), nor could it have been with any piece other than the rook or king. Therefore, White just castled, and thus the white king never moved before that.

Part 3:
Among the white pieces captured by the black pawn from E7 was the white rook from H1. Since White has just castled, and the white king never moved before that, how did the rook from H1 get onto the board to be captured?
The only possible explanation is that the pawns on G3 and H3 cross-captured to let out the rook: the pawn on G3 really came from H2 and vice-versa. Since the pawn on G3 comes from H2, the black bishop on H2 has always been confined to G1 and H2. How did the bishop get there? It must have been promoted.

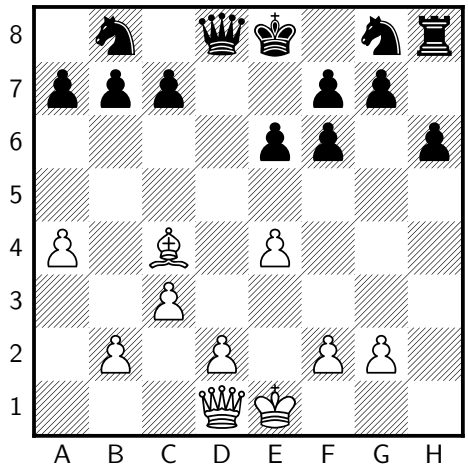
Part 4:
The promoted black bishop on H2 must have been promoted on G1. The pawn which was promoted must have come from G7, since neither of the pawns from F6 or H6 could make a capture to get to the G-file (all six missing white pieces have been accounted for). The Pawn from E7 has promoted to the bishop on A2.
What happened was this: the white pawn from G2 made its capture on H3 while the pawn on G3 was still on H2. This allowed the black pawn to come down and be promoted (after the white rook from H1 got out), and then the pawn on H2 made its capture on G3.

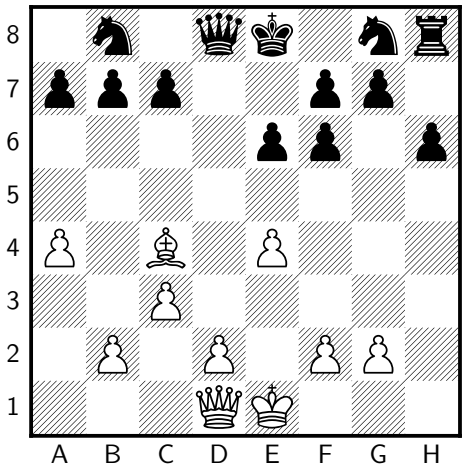
Conclusion:
We already know the mystery piece is black. It can't be a pawn, because we've accounted for all missing black pawns. It can't be a queen or a rook, since there couldn't have been any more promotions by Black. It is therefore a bishop or a knight. However, White has just castled and moved his king over D1, so the mystery piece cannot be a bishop (the king may not cross through check while castling). Therefore, the mystery piece must be **a black knight**.

Part 4: Very difficult problems

Problem 20: The hidden castle
Difficulty: ★★★★★★★★

There is a white castle hidden on this board. Where is it?
None of the royalty has moved or been under attack.



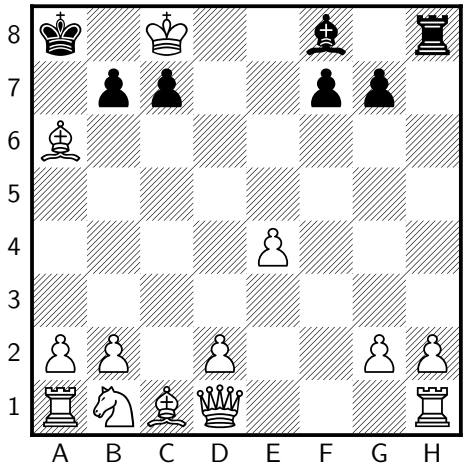


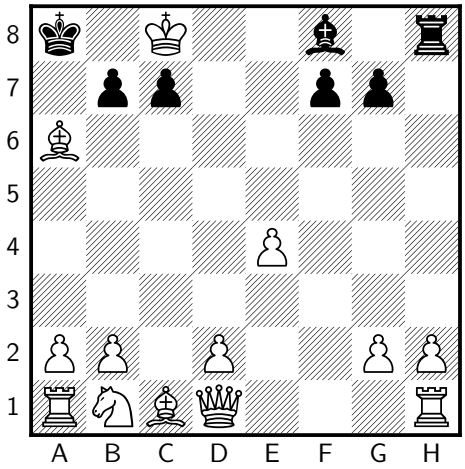
Solution

See "The Hidden Castle" in *The Chess Mysteries of the Arabian Knights*.

Problem 21: Who moved last?
Difficulty: ★★★★★★★★

After many moves of chess, the board looks as follows.
Who moved last?



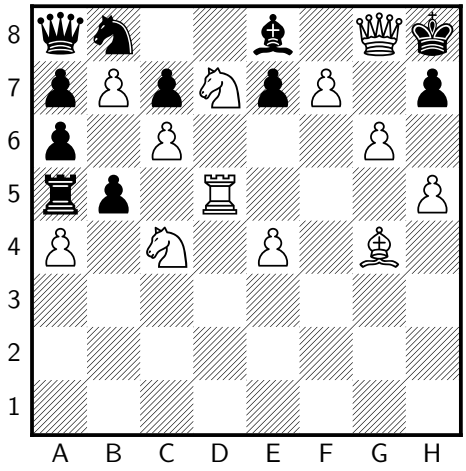


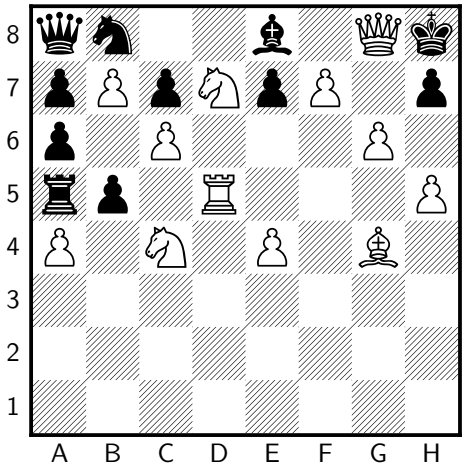
Solution

See "A Vital Decision" in *The Chess Mysteries of the Arabian Knights*.

Problem 22: The king in disguise
Difficulty: ★★★★★★★★

The white king is exploring his kingdom under a disguise. He could look like any piece of any color. Show that he must be on C7.





Solution

Black is in check, so we know that it is Black's move and White is not in check.
Assume the white king is not on C7. Where else could he hide? First, we exclude the black pawns on A6, A7, and B5, since the white king would be in check in any of those positions.

The pawn on A6 came from B7 by capturing one piece, and the pawn on B5 came from D7 by capturing two. (Note that this may not be true if we don't assume the pawn on C7 is real.) We've counted three captures, all on white squares, so the white black-square bishop must have been captured separately.

Thus, at least four white pieces have been captured. White has 12 pieces on the board, so the white king must be disguised as a white piece if he isn't on C7. If we Exclude a few more pieces in check, we now see that the white king must be on D5, E4, G4, or H5 if he isn't on C7.

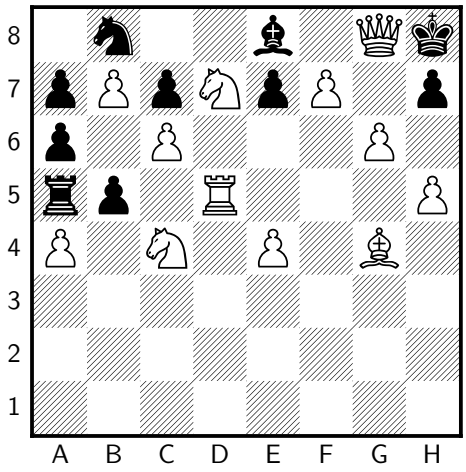
The white queen has to have moved from F8 to capture a piece on G8 to put Black in check. What was Black's move before this? It couldn't have been the king from G7, since the white queen wouldn't have been able to enter F8. It couldn't have been any other piece on the board, since they are all trapped. So, Black's last move must have been with the mystery piece on G8.

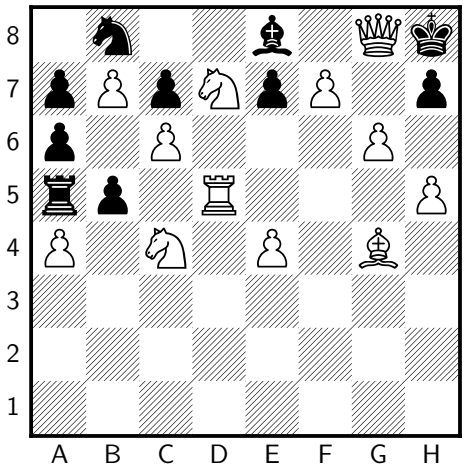
Where did it come from? This piece can't be a bishop (how would it get in?), so it must be a queen, rook, or knight. If it is a queen or rook, it must have come from G7, which is impossible—the white queen wouldn't be able to get in. The mystery piece must therefore be a knight. It couldn't have come from H6 (again, the queen couldn't have gotten in to deliver a check), so it must have come from F6.

We now know that the white king is not on D5, E4, G4, or H5, since all those were in check when the black knight was on F6. However, the white king must be on one of those four squares if he isn't on C7. This is a contradiction — therefore the king must be hiding on C7.

Problem 23: The king in disguise once more
Difficulty: ★★★★★☆☆☆

The white king is again exploring his kingdom, now under a different disguise. Where is he?
Hint: "different disguise" implies that the white king looks like a different piece!





Solution

Use the same arguments as before, but now assume that the king isn't a black pawn.

Again, the king is disguised as a white piece, and must be on D5, E4, G4, H5, or B7.
For the same reasons as above, he can't be on D5, E4, G4, or H5, so he must be on B7.